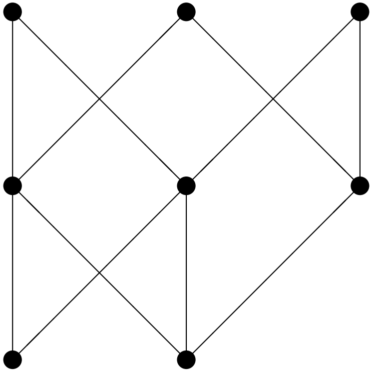
Using Algebraic Topology to Prove Performance Guarantees for a Constraint Satisfaction Algorithm?

Bernd Schröder

Definition.

Definition. *An* **ordered set** *is a pair* $(P, \leq)$ *of a set* P *and a reflexive, antisymmetric and transitive relation* $\leq$*, the order relation.*

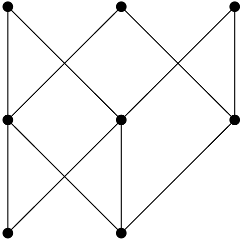
Definition. *An* **ordered set** *is a pair* $(P, \leq)$ *of a set* P *and a reflexive, antisymmetric and transitive relation* $\leq$*, the order relation.*



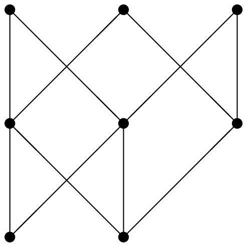
Definition.

Definition. *A function from an ordered set* $(P,\leq)$ *to another ordered set* $(Q,\leq)$ *is called* **order-preserving** *iff, for all* $x,y\in P$ *we have that* $x\leq y$ *implies* $f(x)\leq f(y)$.

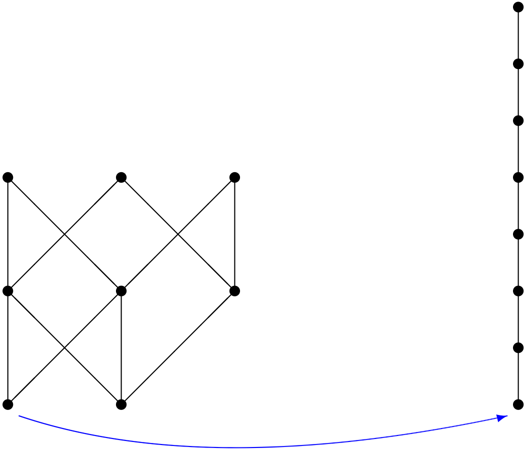
Definition. *A function from an ordered set* $(P, \leq)$ *to another ordered set* $(Q, \leq)$ *is called* **order-preserving** *iff, for all* $x, y \in P$ *we have that* $x \leq y$ *implies* $f(x) \leq f(y)$.



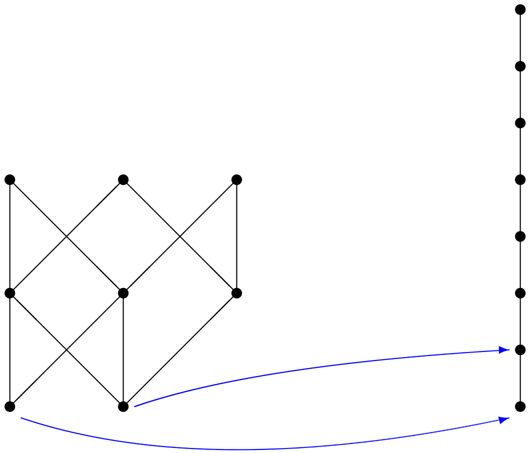
Definition. *A function from an ordered set* $(P, \leq)$ *to another ordered set* $(Q, \leq)$ *is called* **order-preserving** *iff, for all* $x, y \in P$ *we have that* $x \leq y$ *implies* $f(x) \leq f(y)$.



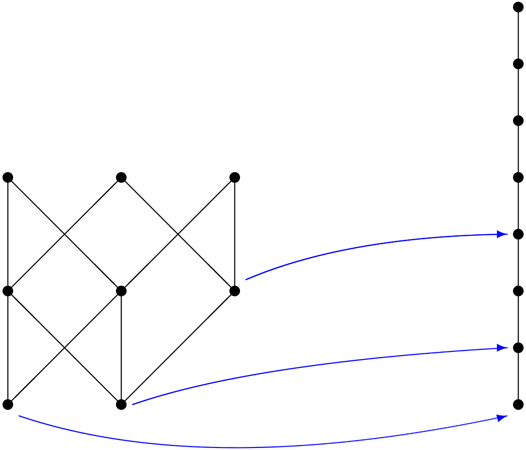
Definition. *A function from an ordered set* $(P,\leq)$ *to another ordered set* $(Q,\leq)$ *is called* **order-preserving** *iff, for all* $x,y\in P$ *we have that* $x\leq y$ *implies* $f(x)\leq f(y)$.



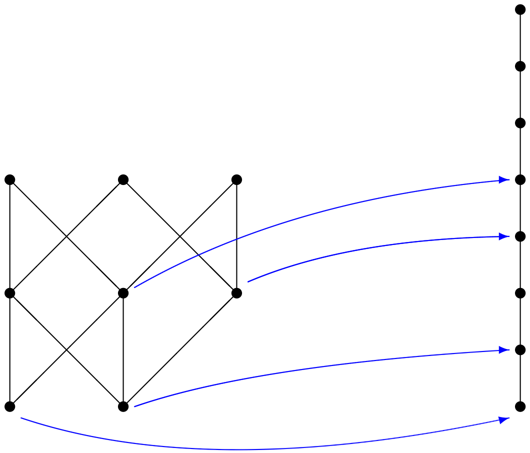
Definition. *A function from an ordered set* $(P, \leq)$ *to another ordered set* $(Q, \leq)$ *is called* **order-preserving** *iff, for all* $x, y \in P$ *we have that* $x \leq y$ *implies* $f(x) \leq f(y)$.



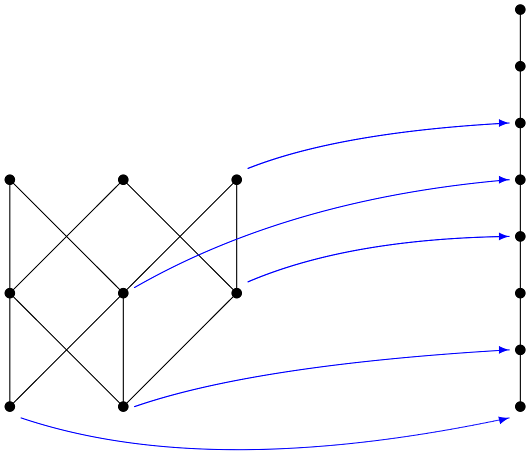
Definition. *A function from an ordered set* $(P, \leq)$ *to another ordered set* $(Q, \leq)$ *is called* **order-preserving** *iff, for all* $x, y \in P$ *we have that* $x \leq y$ *implies* $f(x) \leq f(y)$.



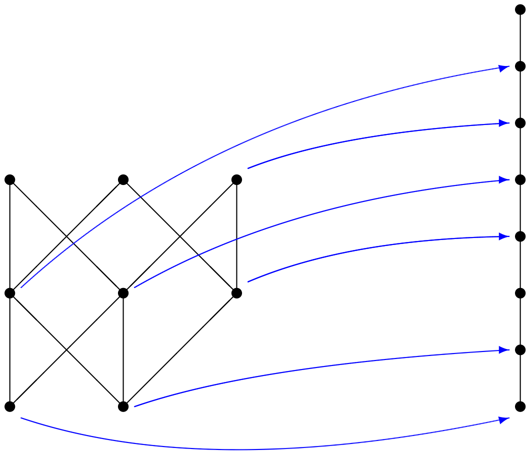
Definition. *A function from an ordered set* $(P,\leq)$ *to another ordered set* $(Q,\leq)$ *is called* **order-preserving** *iff, for all* $x,y\in P$ *we have that* $x\leq y$ *implies* $f(x)\leq f(y)$.



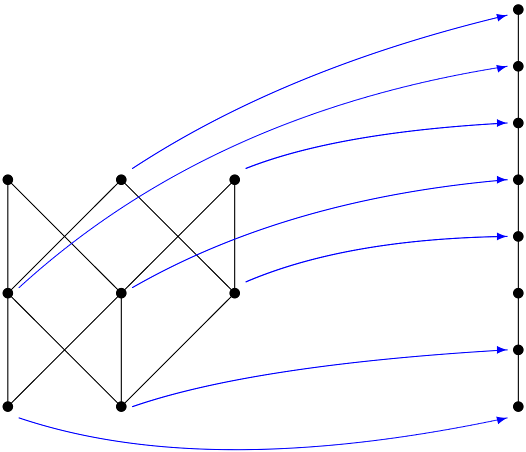
Definition. *A function from an ordered set* $(P, \leq)$ *to another ordered set* $(Q, \leq)$ *is called* **order-preserving** *iff, for all* $x, y \in P$ *we have that* $x \leq y$ *implies* $f(x) \leq f(y)$.



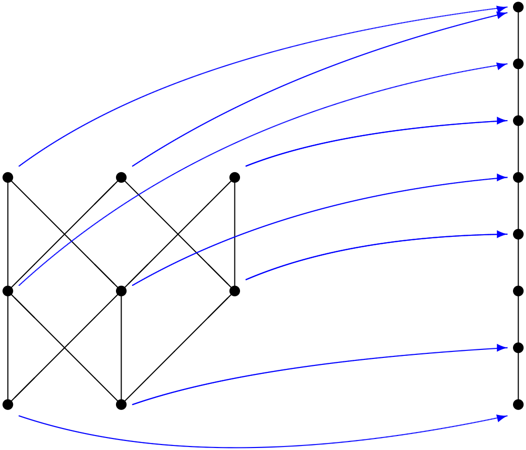
Definition. *A function from an ordered set* $(P, \leq)$ *to another ordered set* $(Q, \leq)$ *is called* **order-preserving** *iff, for all* $x, y \in P$ *we have that* $x \leq y$ *implies* $f(x) \leq f(y)$.



Definition. *A function from an ordered set* $(P,\leq)$ *to another ordered set* $(Q,\leq)$ *is called* **order-preserving** *iff, for all* $x,y\in P$ *we have that* $x\leq y$ *implies* $f(x)\leq f(y)$.



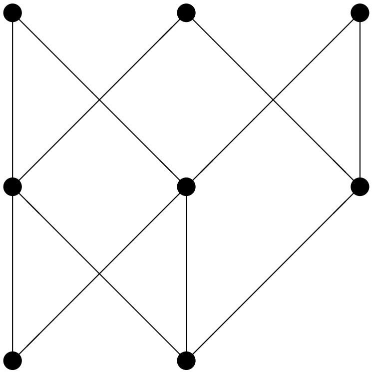
Definition. *A function from an ordered set* $(P,\leq)$ *to another ordered set* $(Q,\leq)$ *is called* **order-preserving** *iff, for all* $x,y\in P$ *we have that* $x\leq y$ *implies* $f(x)\leq f(y)$.



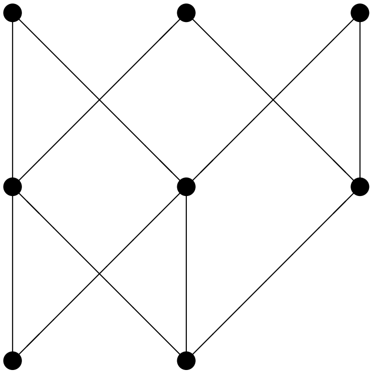
Definition.

Definition. *An ordered set is said to have the* **fixed point property** *iff every order-preserving self-map has a fixed point.*

Definition. *An ordered set is said to have the* **fixed point property** *iff every order-preserving self-map has a fixed point.*



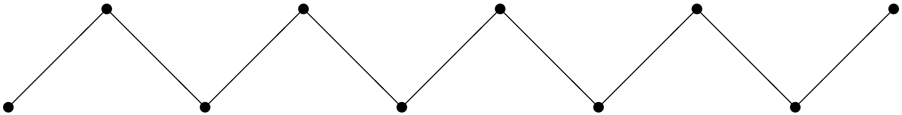
Definition. *An ordered set is said to have the* **fixed point property** *iff every order-preserving self-map has a fixed point.*



... better not yet

Definition. *An ordered set is said to have the* **fixed point property** *iff every order-preserving self-map has a fixed point.*

Definition. *An ordered set is said to have the* **fixed point property** *iff every order-preserving self-map has a fixed point.*

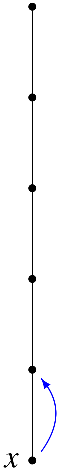


“The Usual Iteration” when $x \leq f(x)$

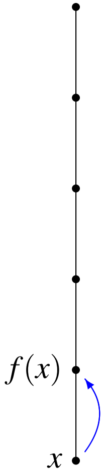
“The Usual Iteration” when $x \leq f(x)$

"The Usual Iteration" when $x \leq f(x)$

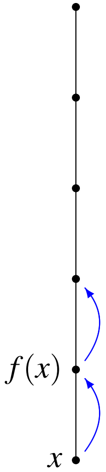
"The Usual Iteration" when $x \leq f(x)$



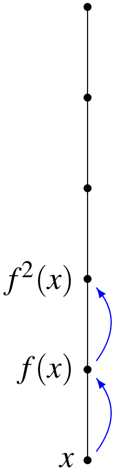
“The Usual Iteration” when $x \leq f(x)$



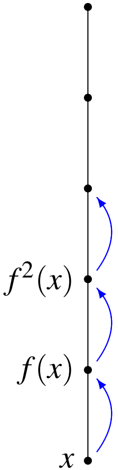
"The Usual Iteration" when $x \leq f(x)$



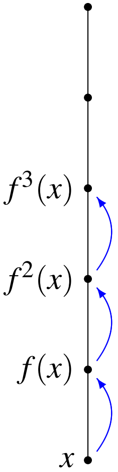
“The Usual Iteration” when $x \leq f(x)$



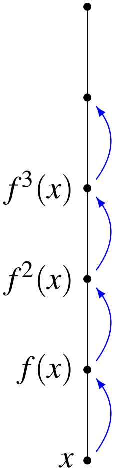
“The Usual Iteration” when $x \leq f(x)$



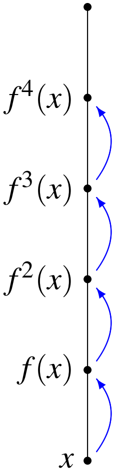
“The Usual Iteration” when $x \leq f(x)$



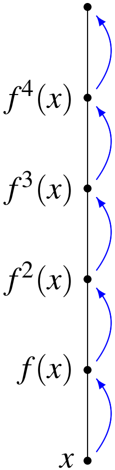
"The Usual Iteration" when $x \leq f(x)$



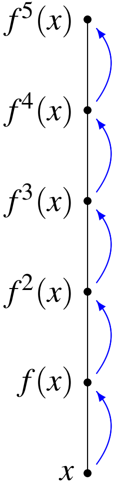
"The Usual Iteration" when $x \leq f(x)$



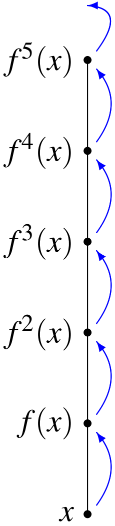
"The Usual Iteration" when $x \leq f(x)$



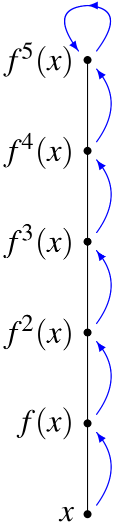
"The Usual Iteration" when $x \leq f(x)$



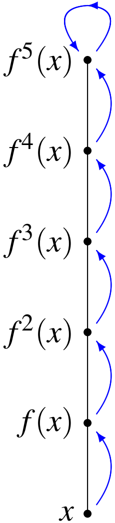
"The Usual Iteration" when $x \leq f(x)$



"The Usual Iteration" when $x \leq f(x)$

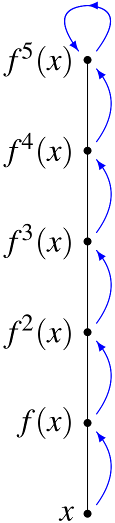


“The Usual Iteration” when $x \leq f(x)$



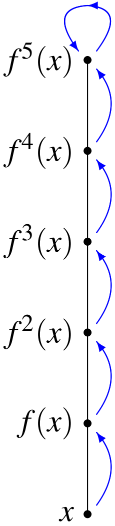
Theorem.

“The Usual Iteration” when $x \leq f(x)$



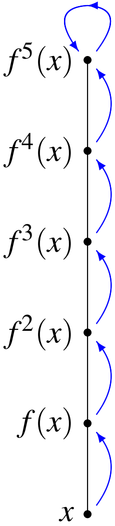
Theorem. *The* **Abian–Brown Theorem**.

"The Usual Iteration" when $x \leq f(x)$



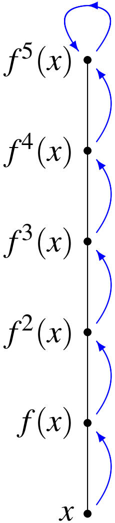
Theorem. *The* **Abian–Brown Theorem**. *Let P be a chain-complete ordered set and let $f : P \to P$ be order-preserving.*

"The Usual Iteration" when $x \leq f(x)$



Theorem. *The* **Abian–Brown Theorem**. *Let P be a chain-complete ordered set and let $f : P \to P$ be order-preserving. If there is an $x \in P$ with $x \leq f(x)$*

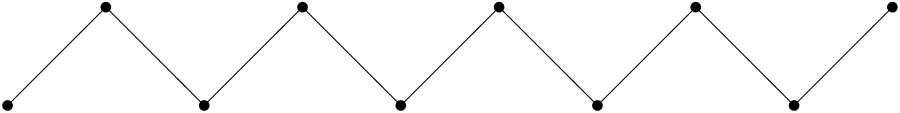
"The Usual Iteration" when $x \leq f(x)$



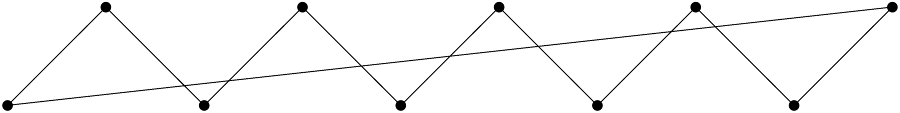
Theorem. *The* **Abian–Brown Theorem**. *Let P be a chain-complete ordered set and let $f : P \to P$ be order-preserving. If there is an $x \in P$ with $x \leq f(x)$, then f has a smallest fixed point above x.*

The “Enemy”

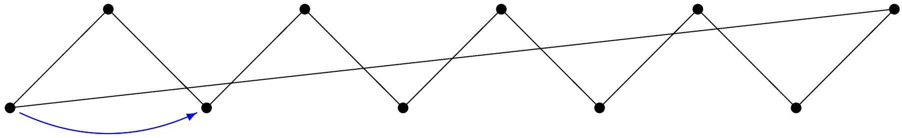
The "Enemy"



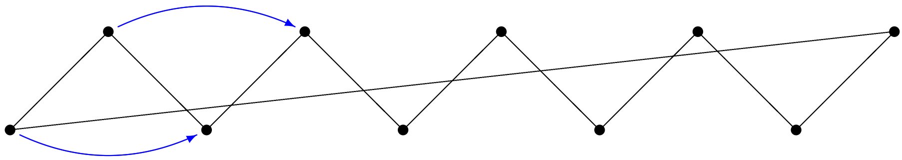
The “Enemy”



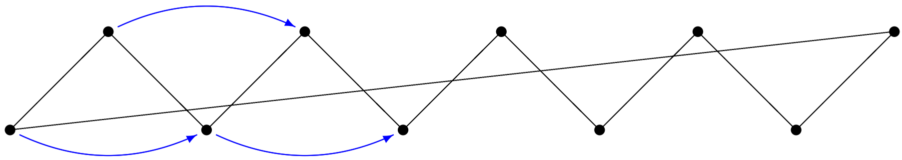
The “Enemy”



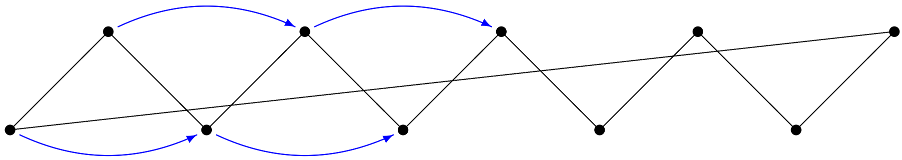
The “Enemy”



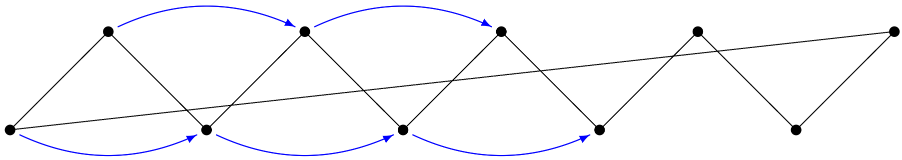
The "Enemy"



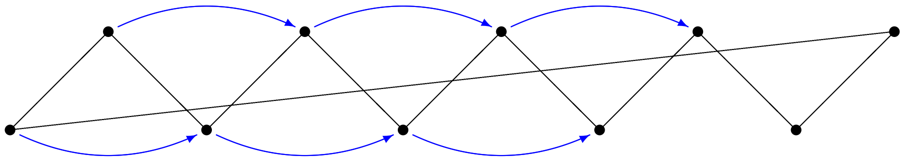
The “Enemy”



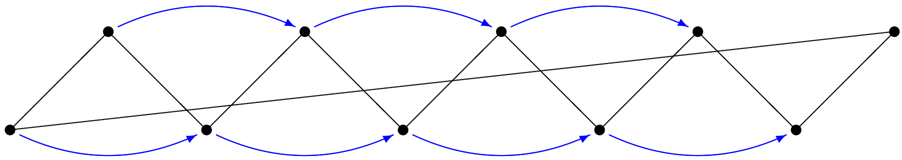
The “Enemy”



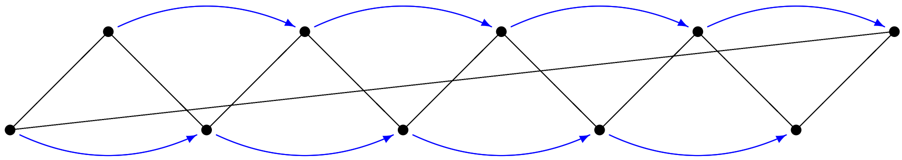
The “Enemy”



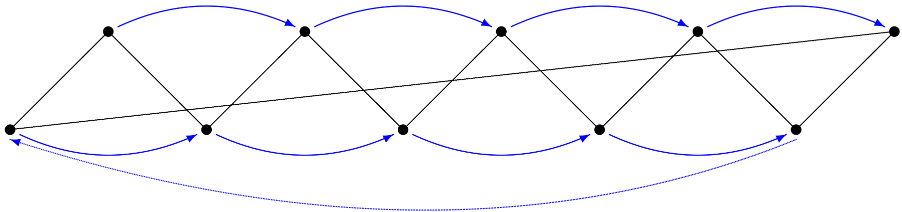
The “Enemy”



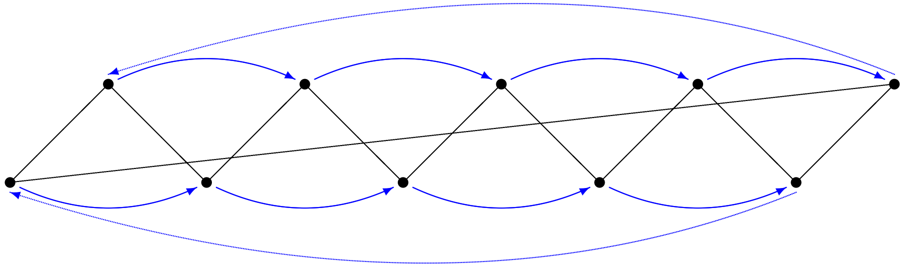
The “Enemy”



The “Enemy”



The "Enemy"



Definition.

Definition. *Let P be an ordered set.*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***).*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem.

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction.*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof.

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving.

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too

Definition. *Let P be an ordered set. Then an order-preserving map $r: P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r: P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r: P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f: r[P] \to r[P]$ be order-preserving. Then $f \circ r: P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$.

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P]$

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$ and hence $x = r(x)$

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$ and hence $x = r(x)$, which means that x

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$ and hence $x = r(x)$, which means that $x = f(r(x))$

Definition. *Let P be an ordered set. Then an order-preserving map $r: P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r: P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r: P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f: r[P] \to r[P]$ be order-preserving. Then $f \circ r: P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$ and hence $x = r(x)$, which means that $x = f(r(x)) = f(x)$

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$ and hence $x = r(x)$, which means that $x = f(r(x)) = f(x)$ is a fixed point of f.

Definition. *Let P be an ordered set. Then an order-preserving map $r : P \to P$ is called a* **retraction** *iff $r^2 = r$ (that is, iff r is* **idempotent***). We will say that $R \subseteq P$ is a* **retract** *of P iff there is a retraction $r : P \to P$ with $r[P] = R$.*

Theorem. *Let P be an ordered set with the fixed point property and let $r : P \to P$ be a retraction. Then $r[P]$ has the fixed point property.*

Proof. Let $f : r[P] \to r[P]$ be order-preserving. Then $f \circ r : P \to P$ is order-preserving, too, and hence it has a fixed point $x = f(r(x))$. But then $x \in f[P] \subseteq r[P]$ and hence $x = r(x)$, which means that $x = f(r(x)) = f(x)$ is a fixed point of f. ∎

Definition.

Definition. *Let* P *be an ordered set and let* $x, y \in P$.

Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x*

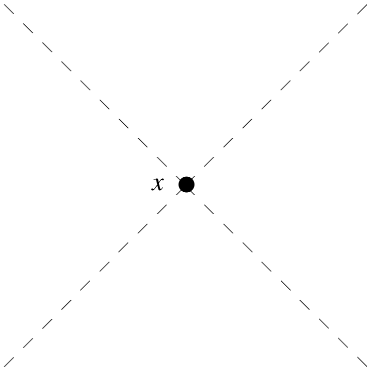
Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y.*

Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*

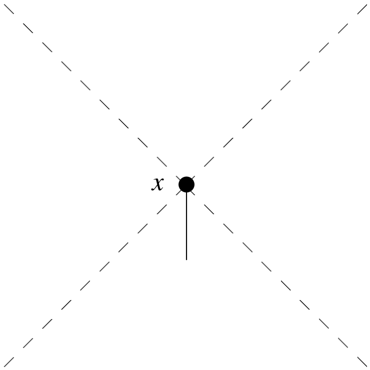
Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*

Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*

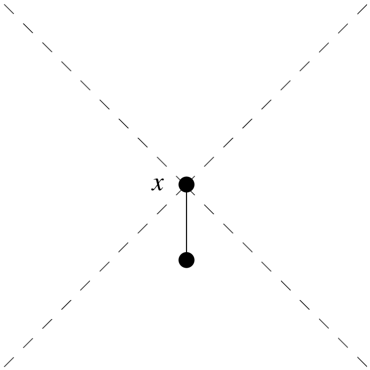
Definition. *Let P be an ordered set and let $x,y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



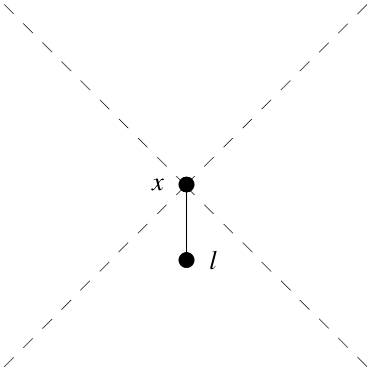
Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



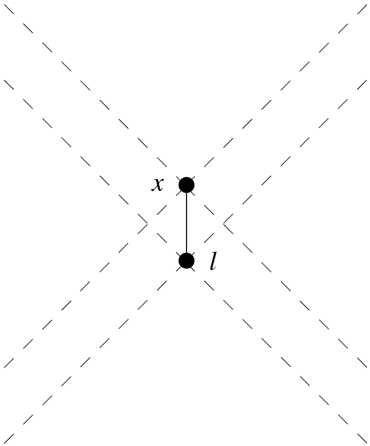
Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



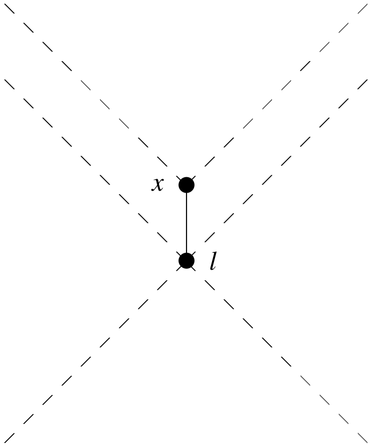
Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



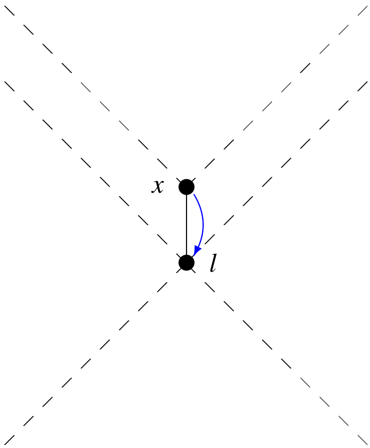
Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



Definition. *Let P be an ordered set and let $x, y \in P$. If $x < y$ and there is no $z \in P$ so that $x < z < y$, then y is called an* **upper cover** *of x and x is called a* **lower cover** *of y. In a finite ordered set, a point is called* **irreducible** *iff it has exactly one upper cover or exactly one lower cover.*



Theorem.

Theorem. *(Rival)*

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible.*

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof.

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:"

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "⇒:" $P \setminus \{x\}$ is a retract.

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "⇒:" $P \setminus \{x\}$ is a retract. "⇐:"

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

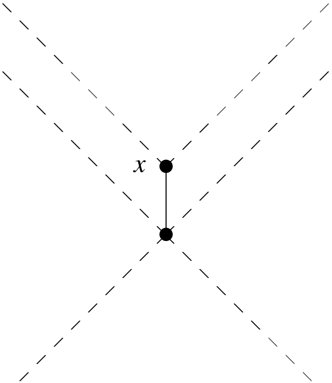
Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving.

Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving. Then $r \circ f|_{P \setminus \{x\}}$ has a fixed point p.

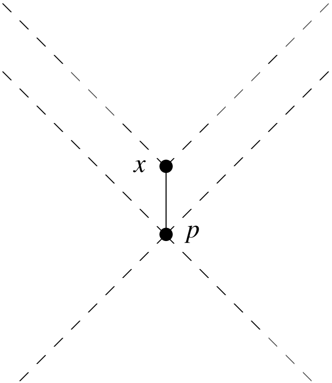
Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving. Then $r \circ f|_{P\setminus\{x\}}$ has a fixed point p.



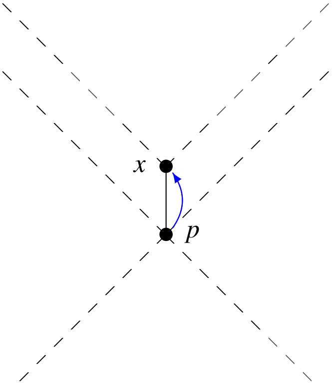
Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving. Then $r \circ f|_{P \setminus \{x\}}$ has a fixed point p.



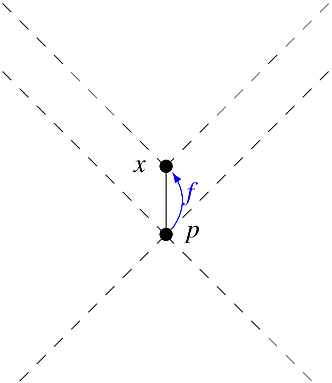
Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving. Then $r \circ f|_{P\setminus\{x\}}$ has a fixed point p.



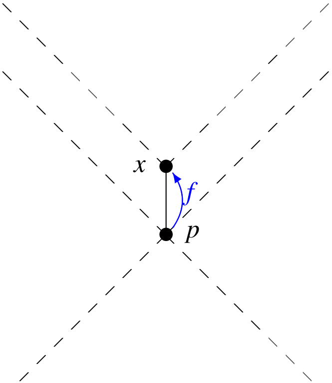
Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving. Then $r \circ f|_{P\setminus\{x\}}$ has a fixed point p.



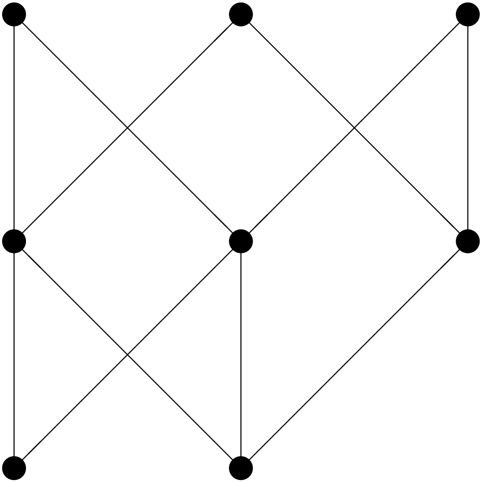
Theorem. *(Rival) Let P be an ordered set and let $x \in P$ be irreducible. Then P has the fixed point property iff $P \setminus \{x\}$ has the fixed point property.*

Proof. "$\Rightarrow$:" $P \setminus \{x\}$ is a retract. "$\Leftarrow$:" Let $f : P \to P$ be order-preserving. Then $r \circ f|_{P \setminus \{x\}}$ has a fixed point p.

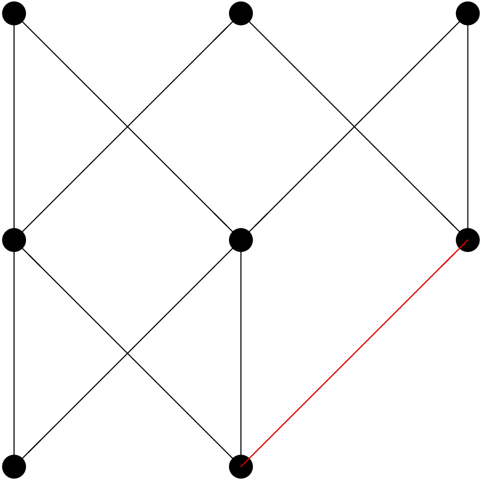


An Example

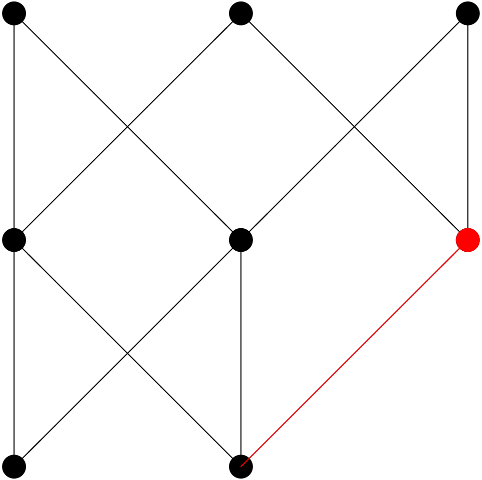
An Example



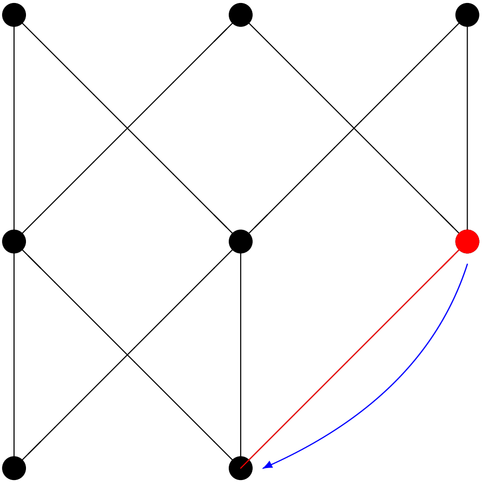
An Example



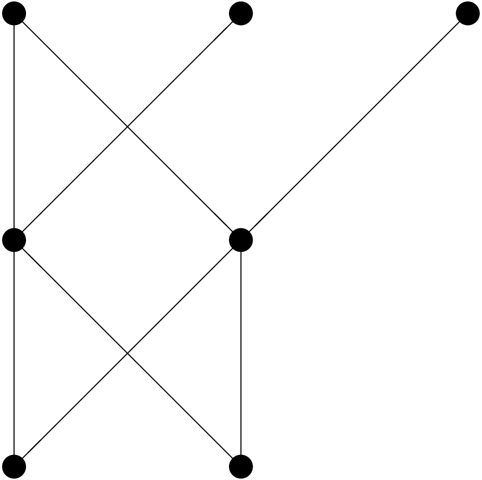
An Example



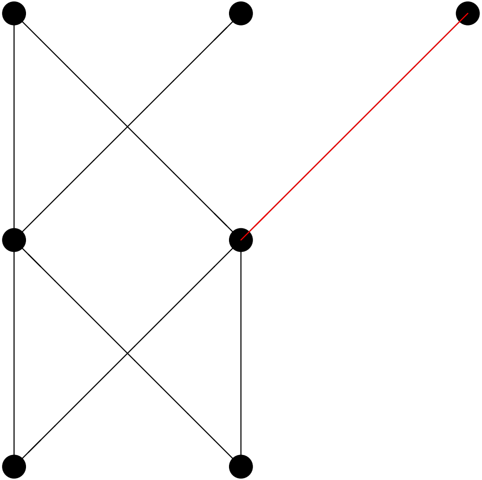
An Example



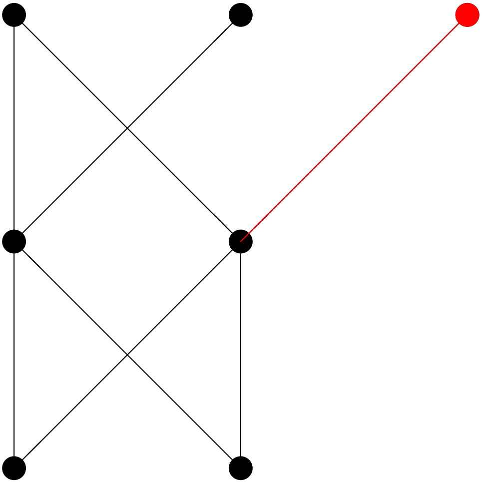
An Example



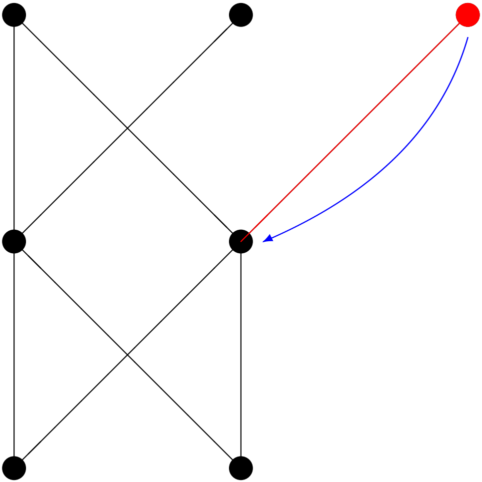
An Example



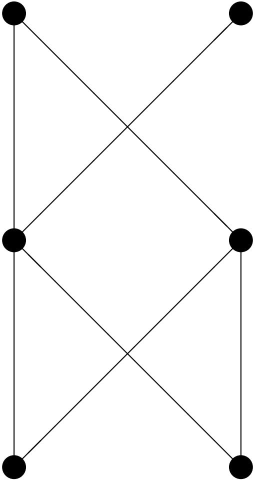
An Example



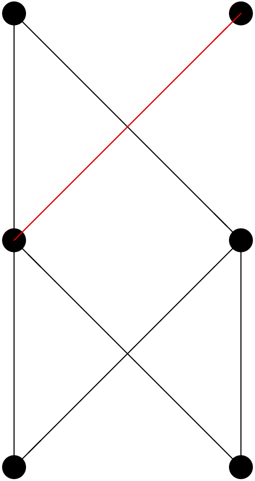
An Example



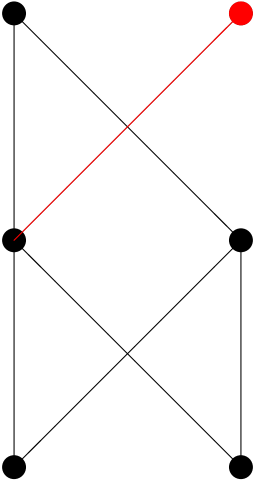
An Example



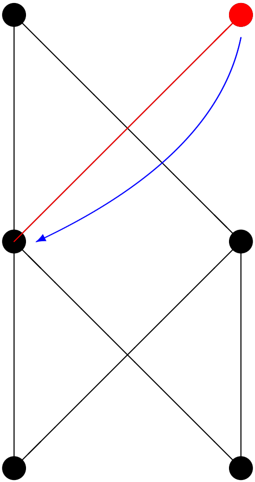
An Example



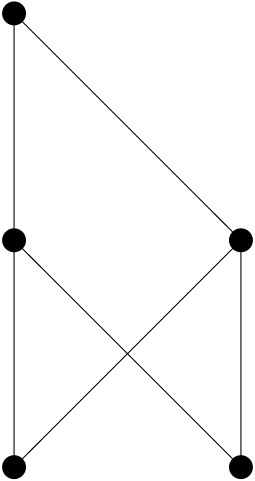
An Example



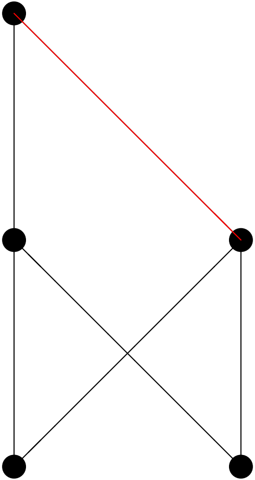
An Example



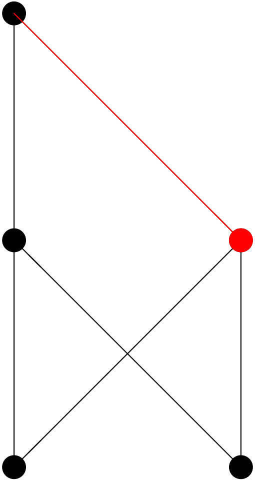
An Example



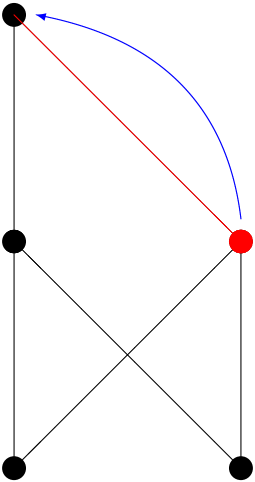
An Example



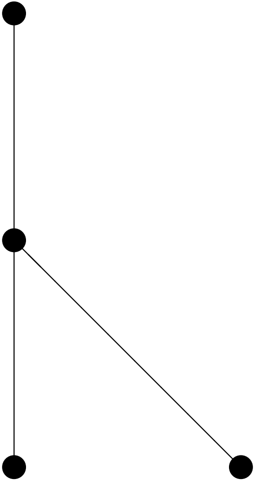
An Example



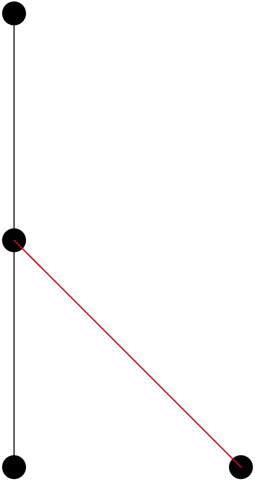
An Example



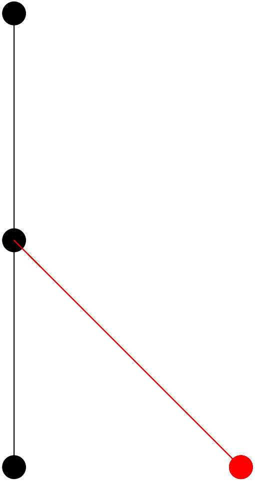
An Example



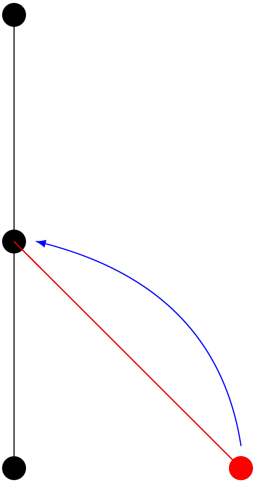
An Example



An Example



An Example

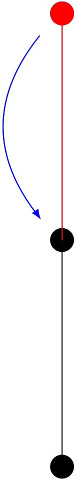


An Example

An Example

An Example

An Example

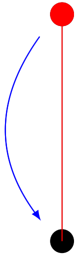


An Example

An Example

An Example

An Example



An Example

How Strong is This Idea?

How Strong is This Idea?

Definition.

How Strong is This Idea?

Definition. An finite ordered set $P = \{p_1, \ldots, p_n\}$ is called **dismantlable**

How Strong is This Idea?

Definition. An finite ordered set $P = \{p_1, \ldots, p_n\}$ is called **dismantlable (by irreducibles)**

How Strong is This Idea?

Definition. An finite ordered set $P = \{p_1, \ldots, p_n\}$ is called **dismantlable (by irreducibles)** iff

How Strong is This Idea?

Definition. An finite ordered set $P = \{p_1, \ldots, p_n\}$ is called **dismantlable (by irreducibles)** iff for all $i \in \{2, \ldots, n\}$ the point p_i is irreducible in $\{p_1, \ldots, p_{i-1}\}$.

How Strong is This Idea?

Definition. An finite ordered set $P = \{p_1, \ldots, p_n\}$ is called **dismantlable (by irreducibles)** iff for all $i \in \{2, \ldots, n\}$ the point p_i is irreducible in $\{p_1, \ldots, p_{i-1}\}$.

- An ordered set of height 1 has the fixed point property iff it is dismantlable.

How Strong is This Idea?

Definition. An finite ordered set $P = \{p_1, \ldots, p_n\}$ is called **dismantlable (by irreducibles)** iff for all $i \in \{2, \ldots, n\}$ the point p_i is irreducible in $\{p_1, \ldots, p_{i-1}\}$.

- An ordered set of height 1 has the fixed point property iff it is dismantlable.
- An ordered set of width 2 has the fixed point property iff it is dismantlable.

The Computational Problem is Hard

The Computational Problem is Hard

1. (Duffus and Goddard)

The Computational Problem is Hard

1. (Duffus and Goddard) The decision problem

   Given. A finite ordered set P.
   Question. Does P have a fixed point free order-preserving self map?

   is NP-complete.

The Computational Problem is Hard

1. (Duffus and Goddard) The decision problem

   Given. A finite ordered set P.
   Question. Does P have a fixed point free order-preserving self map?

   is NP-complete.
2. With all due respect

The Computational Problem is Hard

1. (Duffus and Goddard) The decision problem

   Given. A finite ordered set P.
   Question. Does P have a fixed point free order-preserving self map?

   is NP-complete.
2. With all due respect, so what?

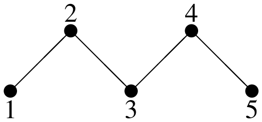
The Computational Problem is Hard

1. (Duffus and Goddard) The decision problem

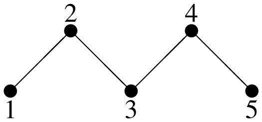
   Given. A finite ordered set P.
   Question. Does P have a fixed point free order-preserving self map?

   is NP-complete.
2. With all due respect, so what? More modestly, which interesting special cases are polynomial?

The 5-fence

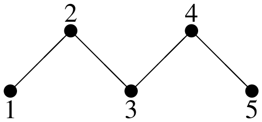


The 5-fence



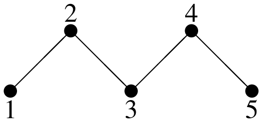
Definition.

The 5-fence



Definition. *A* **binary constraint network** *consists of the following.*

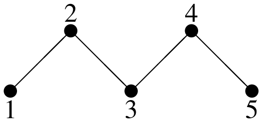
The 5-fence



Definition. *A* **binary constraint network** *consists of the following.*

1. *A set of variables* $x_1, \cdots, x_r$,

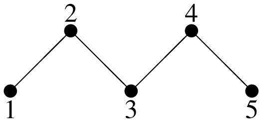
The 5-fence



Definition. *A* **binary constraint network** *consists of the following.*

1. *A set of variables* $x_1, \cdots, x_r$*,*
2. *A set of domains* $D_1, \cdots, D_r$*, one for each variable,*

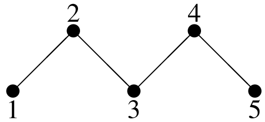
The 5-fence



Definition. *A* **binary constraint network** *consists of the following.*

1. *A set of variables* $x_1, \cdots, x_r$,
2. *A set of domains* $D_1, \cdots, D_r$, *one for each variable,*
3. *A set* $\mathscr{C}$ *of unary and binary constraints.*

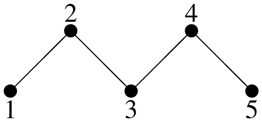
The 5-fence



Definition. *A* **binary constraint network** *consists of the following.*

1. *A set of variables* $x_1, \cdots, x_r$*,*
2. *A set of domains* $D_1, \cdots, D_r$*, one for each variable,*
3. *A set* $\mathscr{C}$ *of unary and binary constraints.*
   - *Each unary constraint consists of a variable* x_i *and a set* $C_i \subseteq D_i$*,*

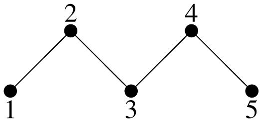
The 5-fence



Definition. *A* **binary constraint network** *consists of the following.*

1. *A set of variables* $x_1, \cdots, x_r$,
2. *A set of domains* $D_1, \cdots, D_r$, *one for each variable,*
3. *A set* $\mathscr{C}$ *of unary and binary constraints.*
   - *Each unary constraint consists of a variable* x_i *and a set* $C_i \subseteq D_i$,
   - *Each binary constraint consists of a set of two variables* $\{x_i, x_j\}$ *and a binary relation* $C_{ij} \subseteq D_i \times D_j$,

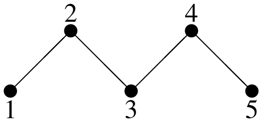
The 5-fence



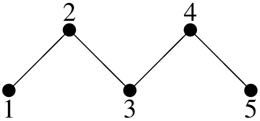
Definition. *A* **binary constraint network** *consists of the following.*

1. *A set of variables $x_1, \cdots, x_r$,*
2. *A set of domains $D_1, \cdots, D_r$, one for each variable,*
3. *A set $\mathscr{C}$ of unary and binary constraints.*
   - *Each unary constraint consists of a variable x_i and a set $C_i \subseteq D_i$,*
   - *Each binary constraint consists of a set of two variables $\{x_i, x_j\}$ and a binary relation $C_{ij} \subseteq D_i \times D_j$,*
   - *For each set of variables we have at most one constraint.*

The 5-fence

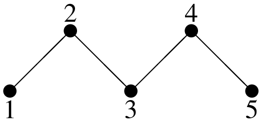


The 5-fence



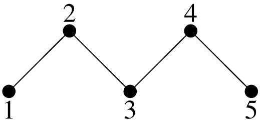
Definition.

The 5-fence



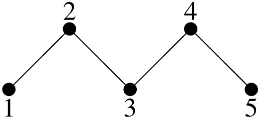
Definition. *For a given constraint network, let* $Y \subseteq \{1, \cdots, n\}$.

The 5-fence



Definition. *For a given constraint network, let* $Y \subseteq \{1, \cdots, n\}$. *Any element* $a \in \prod_{j \in Y} D_j$ *is an* **instantiation** *of the variables in* Y.

The 5-fence



Definition. *For a given constraint network, let* $Y \subseteq \{1, \cdots, n\}$*. Any element* $a \in \prod_{j \in Y} D_j$ *is an* **instantiation** *of the variables in* Y*. An instantiation is called* **consistent** *iff for all* $i, j \in Y$ *we have that* $a_i \in C_i$ *and* $(a_i, a_j) \in C_{ij}$*.*

Definition.

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$.*

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is* $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$.

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

- *If $x_i \not\leq x_j$ and $x_i \not\geq x_j$, there is no constraint between x_i and x_j,*

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

- *If $x_i \not\leq x_j$ and $x_i \not\geq x_j$, there is no constraint between x_i and x_j,*
- *If $x_i \leq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \leq u_j\}$,*

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

- *If $x_i \not\leq x_j$ and $x_i \not\geq x_j$, there is no constraint between x_i and x_j,*
- *If $x_i \leq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \leq u_j\}$,*
- *If $x_i \geq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \geq u_j\}$.*

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

- *If $x_i \not\leq x_j$ and $x_i \not\geq x_j$, there is no constraint between x_i and x_j,*
- *If $x_i \leq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \leq u_j\}$,*
- *If $x_i \geq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \geq u_j\}$.*

If x_i is related to x_j, then "$C_{ij} = C_{ji}$" and so we have exactly one constraint for each set of two variables.

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

- *If $x_i \not\leq x_j$ and $x_i \not\geq x_j$, there is no constraint between x_i and x_j,*
- *If $x_i \leq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \leq u_j\}$,*
- *If $x_i \geq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \geq u_j\}$.*

If x_i is related to x_j, then "$C_{ij} = C_{ji}$" and so we have exactly one constraint for each set of two variables. Any consistent instantiation $(u_{i_1}, \cdots, u_{i_k})$ of k variables $(x_{i_1}, \cdots, x_{i_k})$ corresponds to an order-preserving partial *map from $\{x_{i_1}, \cdots, x_{i_k}\}$ to P*

Definition. *For determining if the ordered set P has a fixed-point-free self-map, we would let our variable set be equal to P, so let $\{x_1, \cdots, x_n\} := P$. For each x_i the domain D_i is $D_i := \{p \in P : p \not\leq x_i \text{ and } p \not\geq x_i\}$. The constraints C_{ij} are*

- *If $x_i \not\leq x_j$ and $x_i \not\geq x_j$, there is no constraint between x_i and x_j,*
- *If $x_i \leq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \leq u_j\}$,*
- *If $x_i \geq x_j$, let C_{ij} be the constraint $C_{ij} := \{(u_i, u_j) \in D_i \times D_j : u_i \geq u_j\}$.*

If x_i is related to x_j, then "$C_{ij} = C_{ji}$" and so we have exactly one constraint for each set of two variables. Any consistent instantiation $(u_{i_1}, \cdots, u_{i_k})$ of k variables $(x_{i_1}, \cdots, x_{i_k})$ corresponds to an order-preserving partial *map from $\{x_{i_1}, \cdots, x_{i_k}\}$ to P, namely the map that maps each x_{i_j} to u_{i_j}.*

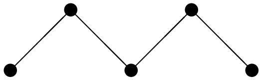
Backtracking

Backtracking

The 5-fence

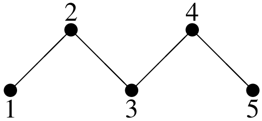
Backtracking

The 5-fence



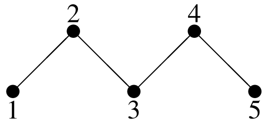
Backtracking

The 5-fence



Backtracking

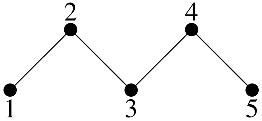
The 5-fence



FPP backtracking tree

Backtracking

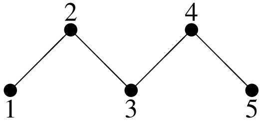
The 5-fence



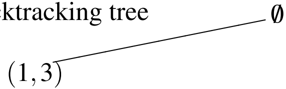
FPP backtracking tree $\emptyset$

Backtracking

The 5-fence

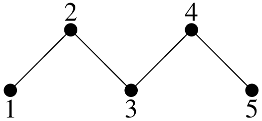


FPP backtracking tree

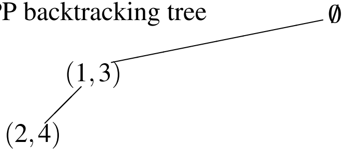


Backtracking

The 5-fence

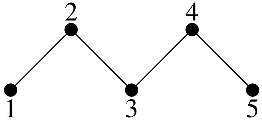


FPP backtracking tree

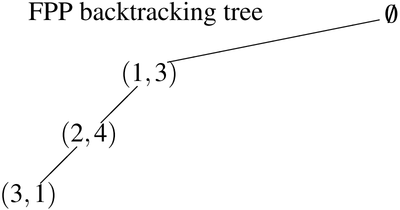


Backtracking

The 5-fence

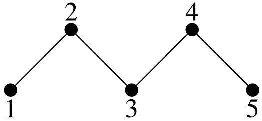


FPP backtracking tree

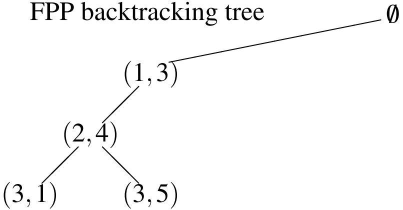


Backtracking

The 5-fence

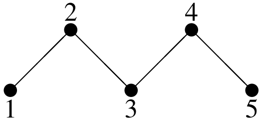


FPP backtracking tree

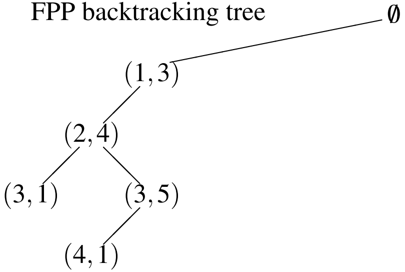


Backtracking

The 5-fence

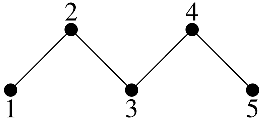


FPP backtracking tree

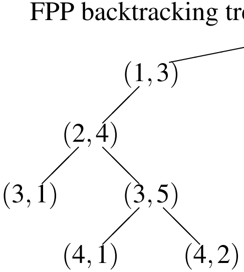


Backtracking

The 5-fence

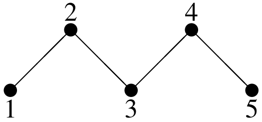


FPP backtracking tree

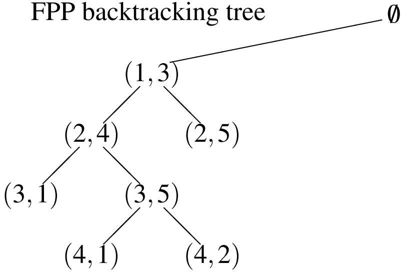


Backtracking

The 5-fence

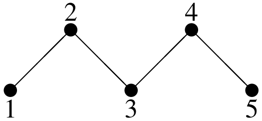


FPP backtracking tree

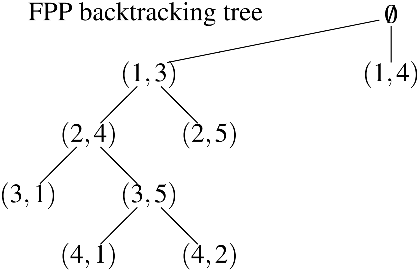


Backtracking

The 5-fence

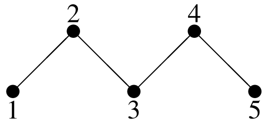


FPP backtracking tree

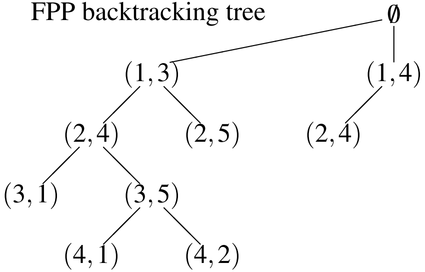


Backtracking

The 5-fence

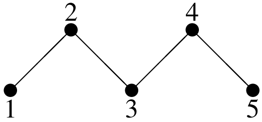


FPP backtracking tree

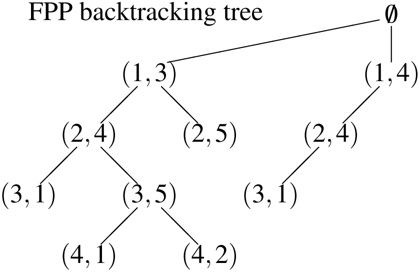


Backtracking

The 5-fence

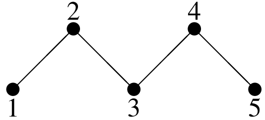


FPP backtracking tree

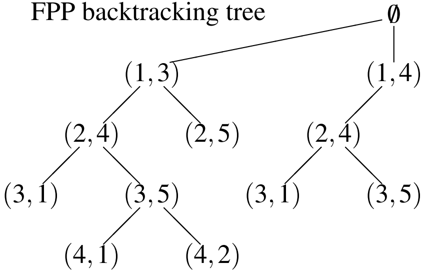


Backtracking

The 5-fence

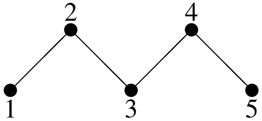


FPP backtracking tree

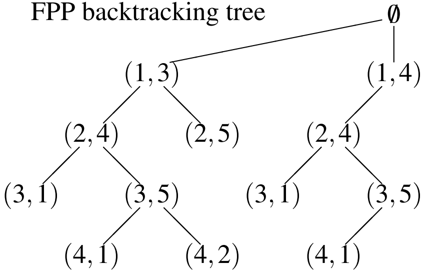


Backtracking

The 5-fence

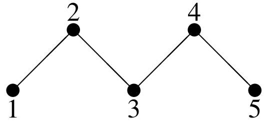


FPP backtracking tree

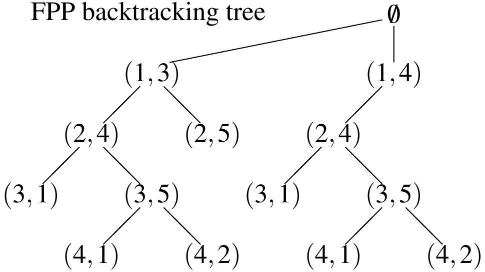


Backtracking

The 5-fence

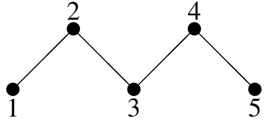


FPP backtracking tree

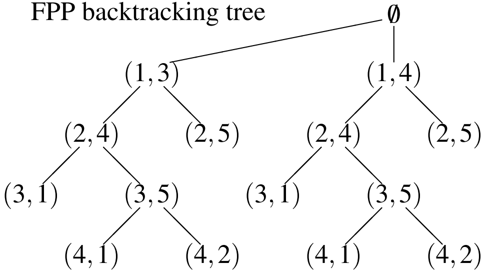


Backtracking

The 5-fence

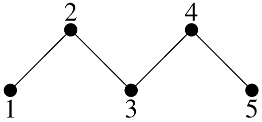


FPP backtracking tree

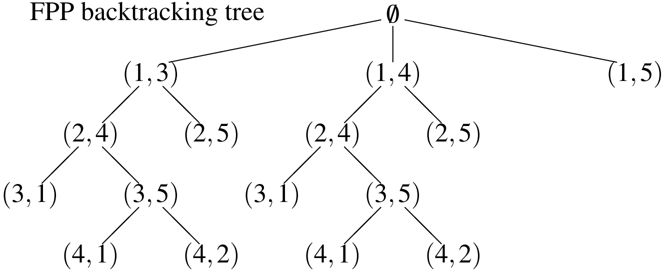


Backtracking

The 5-fence

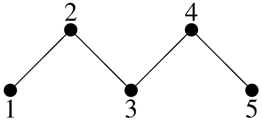


FPP backtracking tree

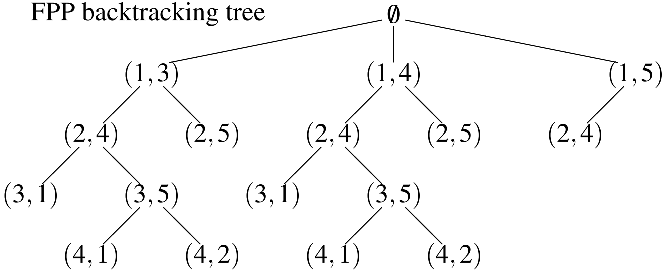


Backtracking

The 5-fence

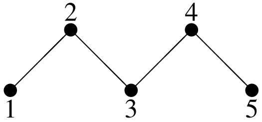


FPP backtracking tree

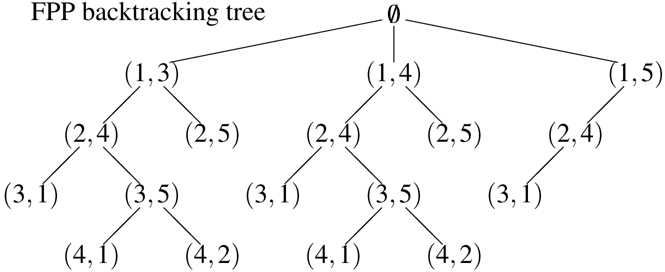


Backtracking

The 5-fence

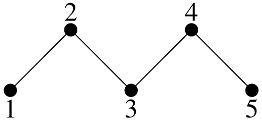


FPP backtracking tree

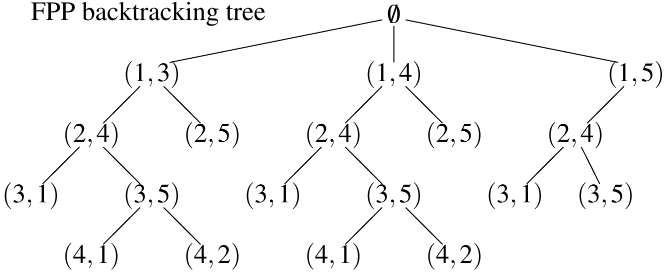


Backtracking

The 5-fence

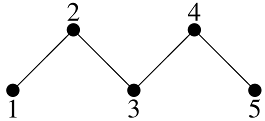


FPP backtracking tree

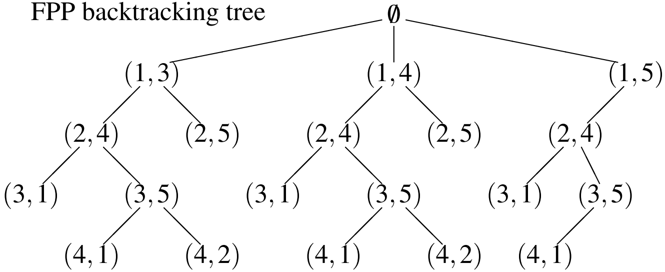


Backtracking

The 5-fence

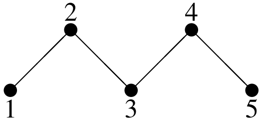


FPP backtracking tree

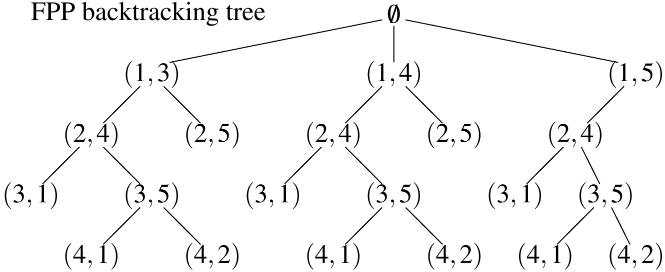


Backtracking

The 5-fence

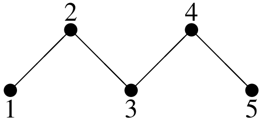


FPP backtracking tree

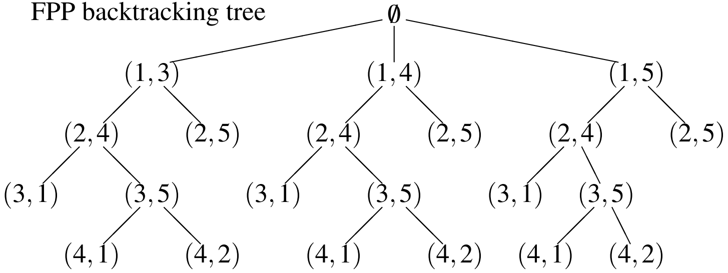


Backtracking

The 5-fence

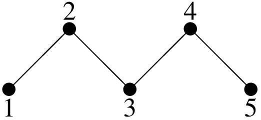


FPP backtracking tree

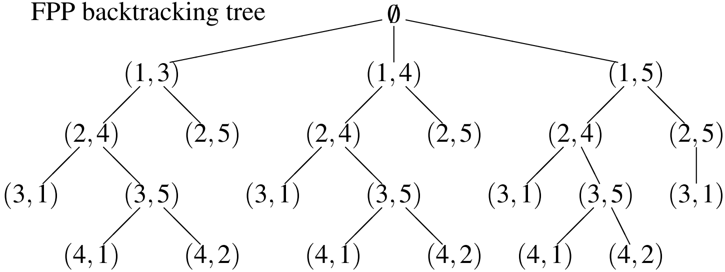


Backtracking

The 5-fence

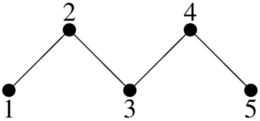


FPP backtracking tree

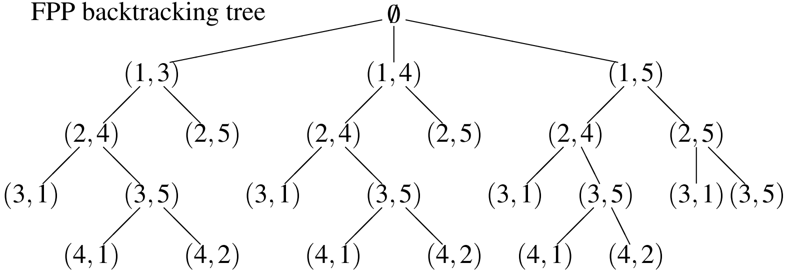


Backtracking

The 5-fence

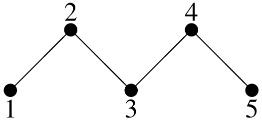


FPP backtracking tree

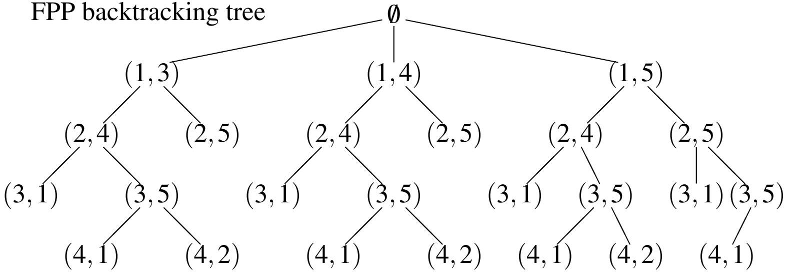


Backtracking

The 5-fence

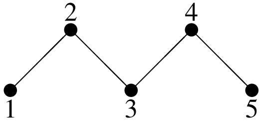


FPP backtracking tree

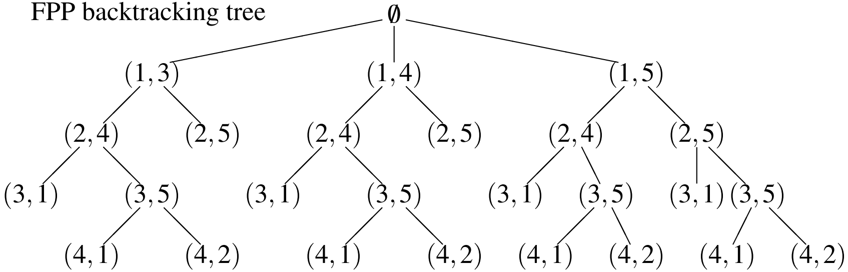


Backtracking

The 5-fence



FPP backtracking tree



Definition.

Definition. *A constraint network is called* k**-consistent** *iff*

Definition. *A constraint network is called* k-**consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables*

Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable*

Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable, there is an instantiation of the* k^{th} *variable such that the* k *instantiations together also are consistent.*

Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable, there is an instantiation of the* k^{th} *variable such that the* k *instantiations together also are consistent.*

In particular

Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable, there is an instantiation of the* k^{th} *variable such that the* k *instantiations together also are consistent.*

In particular
1-*consistency is also called* **node consistency**,

Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable, there is an instantiation of the* k^{th} *variable such that the* k *instantiations together also are consistent.*

In particular
1-*consistency is also called* **node consistency**,
2-*consistency is also called* **arc consistency**,

Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable, there is an instantiation of the* k^{th} *variable such that the* k *instantiations together also are consistent.*

In particular
1-*consistency is also called* **node consistency**,
2-*consistency is also called* **arc consistency**,
3-*consistency is also called* **path consistency**.

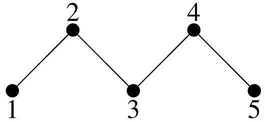
Definition. *A constraint network is called* k**-consistent** *iff, for each consistent instantiation of* $(k-1)$ *variables and any choice of a* k^{th} *variable, there is an instantiation of the* k^{th} *variable such that the* k *instantiations together also are consistent.*

In particular
1-*consistency is also called* **node consistency**,
2-*consistency is also called* **arc consistency**,
3-*consistency is also called* **path consistency**.

A constraint network is **strongly** k**-consistent** *iff for all* $1 \leq j \leq k$ *the network is* j*-consistent.*

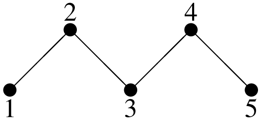
Enforcing Path Consistency

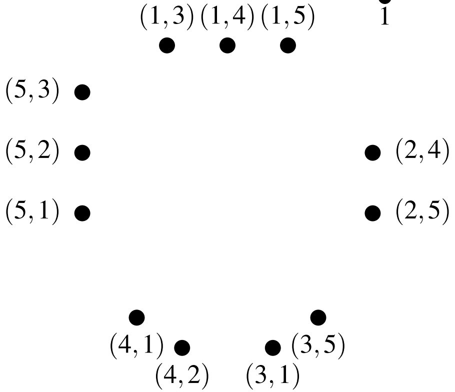
The 5-fence



Enforcing Path Consistency

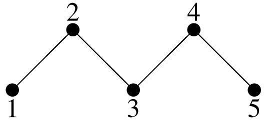
The 5-fence

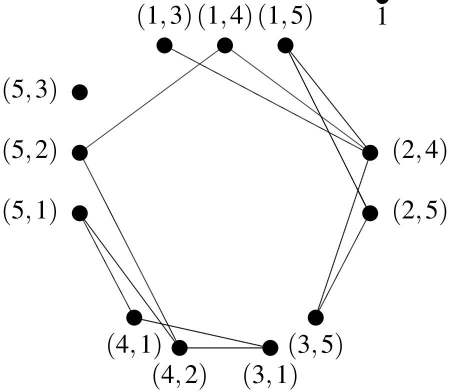




Enforcing Path Consistency

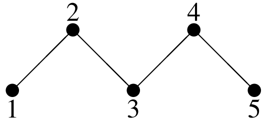
The 5-fence

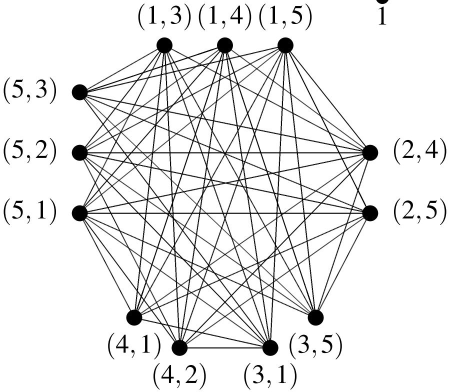




Enforcing Path Consistency

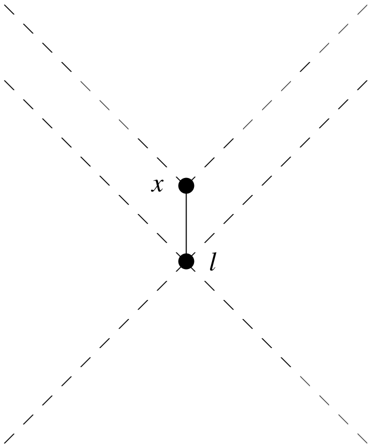
The 5-fence



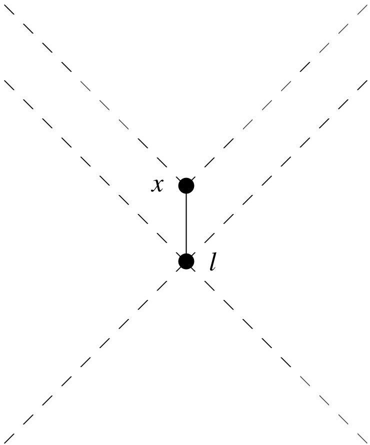


Irreducible Points are Irrelevant for Path Consistency

Irreducible Points are Irrelevant for Path Consistency

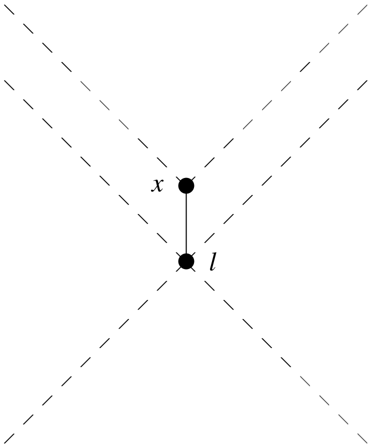


Irreducible Points are Irrelevant for Path Consistency



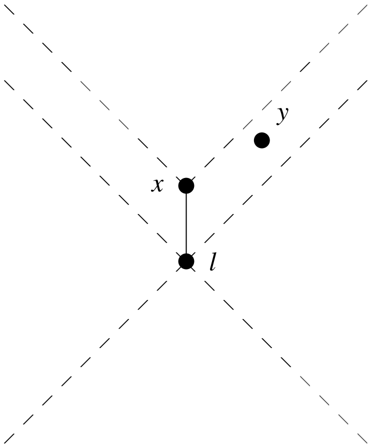
Let x have a unique lower cover l.

Irreducible Points are Irrelevant for Path Consistency



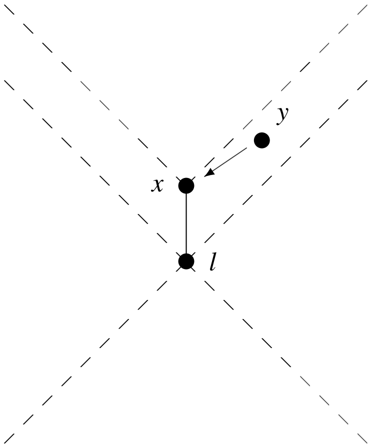
Let x have a unique lower cover l. Let $y > l$ and $y \not> x$.

Irreducible Points are Irrelevant for Path Consistency



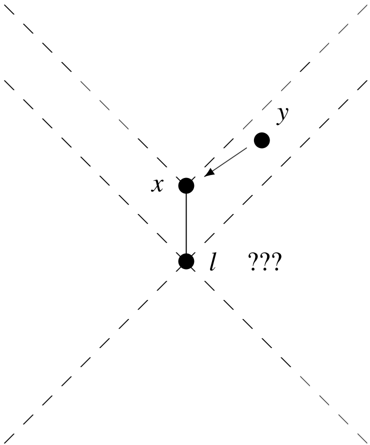
Let x have a unique lower cover l. Let $y > l$ and $y \not> x$.

Irreducible Points are Irrelevant for Path Consistency



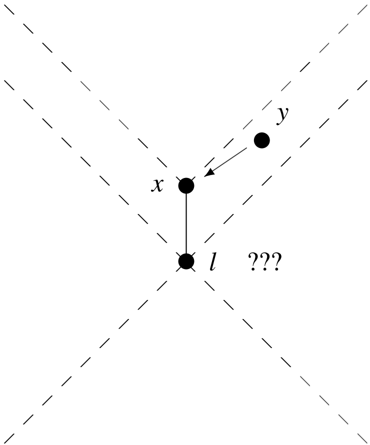
Let x have a unique lower cover l. Let $y > l$ and $y \not> x$.

Irreducible Points are Irrelevant for Path Consistency



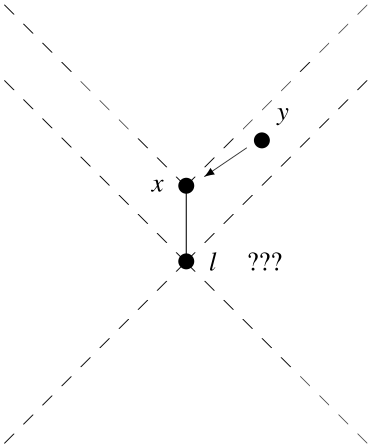
Let x have a unique lower cover l. Let $y > l$ and $y \not> x$.

Irreducible Points are Irrelevant for Path Consistency



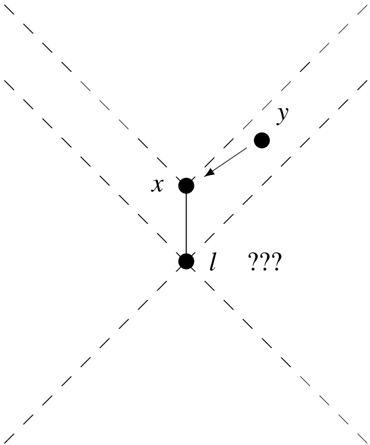
Let x have a unique lower cover l. Let $y > l$ and $y \not> x$. Then (y, x) is not consistent with any instantiation $(l, \cdot)$

Irreducible Points are Irrelevant for Path Consistency

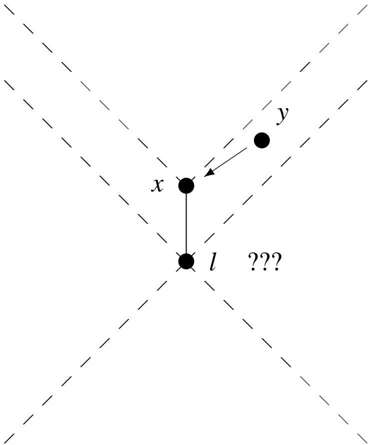


Let x have a unique lower cover l. Let $y > l$ and $y \not> x$. Then (y,x) is not consistent with any instantiation $(l,\cdot)$, so all edges incident with (y,x) can be eliminated.

Irreducible Points are Irrelevant for Path Consistency

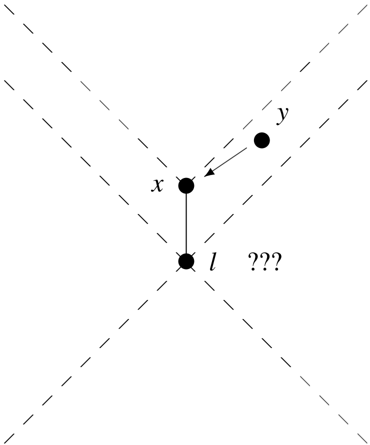


Irreducible Points are Irrelevant for Path Consistency



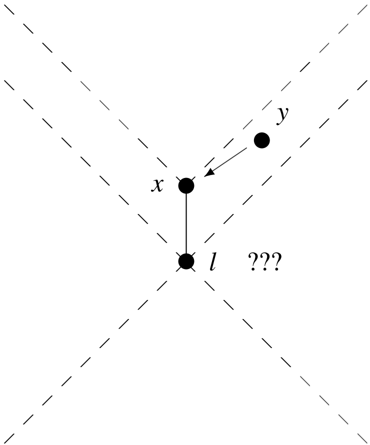
For all other (z,x), we have that, if $\{(a,b),(z,x)\}$ is an edge

Irreducible Points are Irrelevant for Path Consistency



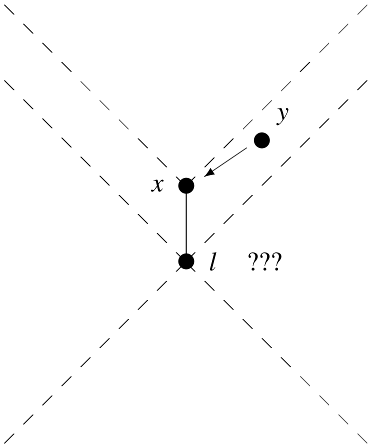
For all other (z,x), we have that, if $\{(a,b),(z,x)\}$ is an edge, then so is $\{(a,b),(z,l)\}$.

Irreducible Points are Irrelevant for Path Consistency



For all other (z,x), we have that, if $\{(a,b),(z,x)\}$ is an edge, then so is $\{(a,b),(z,l)\}$. Hence, (some details omitted) enforcing consistency on $EXPFPF(P)$ yields an empty network

Irreducible Points are Irrelevant for Path Consistency



For all other (z,x), we have that, if $\{(a,b),(z,x)\}$ is an edge, then so is $\{(a,b),(z,l)\}$. Hence, (some details omitted) enforcing consistency on $EXPFPF(P)$ yields an empty network iff enforcing consistency on $EXPFPF(P\setminus\{x\})$ yields an empty network.

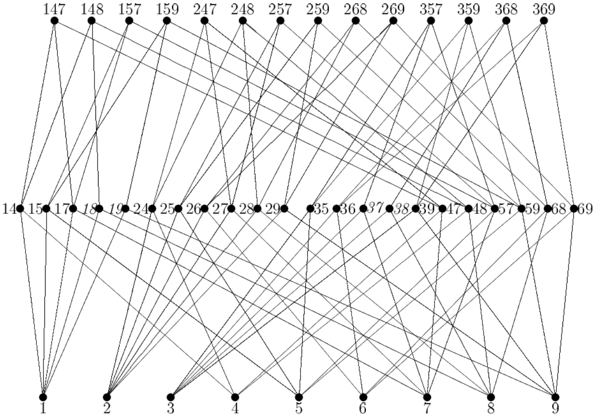
Open Question

Open Question

How strong is path consistency?

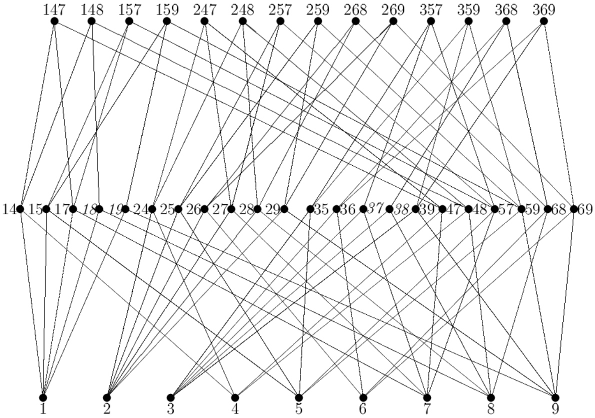
Open Question

How strong is path consistency?



Open Question

How strong is path consistency?



Program

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example.

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

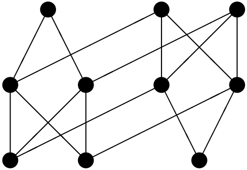
1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property.

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

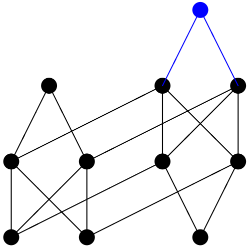
Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?



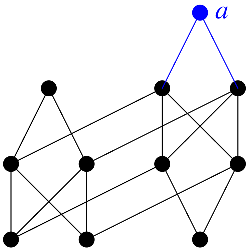
Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?



Open Question

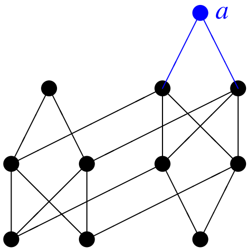
Can we get a similar result for removal of points with an acyclic neighborhood?



Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

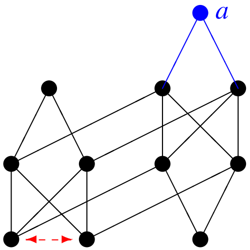
$P2^a$



Open Question

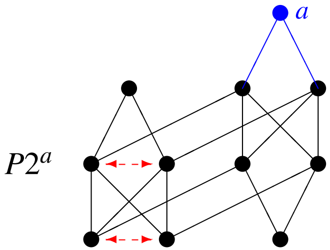
Can we get a similar result for removal of points with an acyclic neighborhood?

$P2^a$



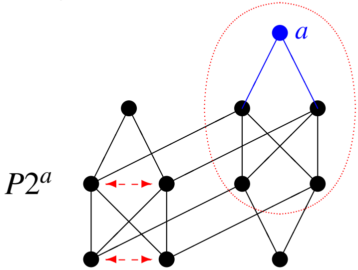
Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?



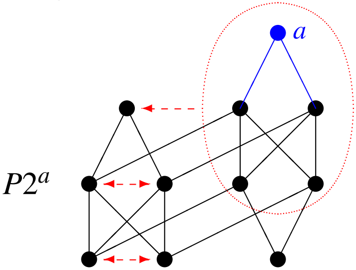
Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?



Open Question

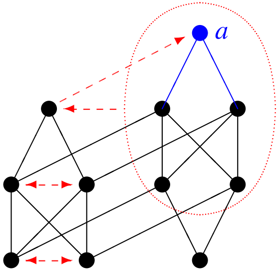
Can we get a similar result for removal of points with an acyclic neighborhood?



Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

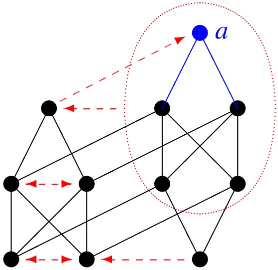
$P2^a$



Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

$P2^a$



Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property.

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property. ($P \setminus \{a\}$ is not acyclic.)

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property. ($P \setminus \{a\}$ is not acyclic.)
2. Middle ground

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property. ($P \setminus \{a\}$ is not acyclic.)
2. Middle ground: Dimension 2 and fpp implies that enforcing path consistency produces an empty expanded (fpp) constraint network.

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property. ($P \setminus \{a\}$ is not acyclic.)
2. Middle ground: Dimension 2 and fpp implies that enforcing path consistency produces an empty expanded (fpp) constraint network. The argument is combinatorial

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property. ($P \setminus \{a\}$ is not acyclic.)
2. Middle ground: Dimension 2 and fpp implies that enforcing path consistency produces an empty expanded (fpp) constraint network. The argument is combinatorial (but not “pointwise”).

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

1. Negative example. There is an ordered set P with a point a for which enforcing path consistency produces an empty expanded (fpp) constraint network for $P \setminus \{a\}$ enforcing arc consistency produces an empty expanded (fpp) constraint network for $(\uparrow a \cup \downarrow a) \setminus \{a\}$ and yet P does not have the fixed point property. ($P \setminus \{a\}$ is not acyclic.)
2. Middle ground: Dimension 2 and fpp implies that enforcing path consistency produces an empty expanded (fpp) constraint network. The argument is combinatorial (but not "pointwise"). Does "collapsility by retractable points" (and fpp) imply that enforcing path consistency produces an empty expanded (fpp) constraint network?

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

Open Question

Can we get a similar result for removal of points with an acyclic neighborhood?

Let's consider the topological side now.

Definition.

Definition. *A* **simplicial complex** *is a pair $K = (V, \mathscr{S})$ of a set V of vertices and a set $\mathscr{S}$ of nonempty subsets of V called* **simplices** *(singular:* **simplex***) such that the following hold.*

Definition. *A* **simplicial complex** *is a pair* $K = (V, \mathscr{S})$ *of a set* V *of vertices and a set* $\mathscr{S}$ *of nonempty subsets of* V *called* **simplices** *(singular:* **simplex***) such that the following hold.*

1. *Every singleton subset of* V *is a simplex.*

Definition. *A* **simplicial complex** *is a pair* $K = (V, \mathscr{S})$ *of a set* V *of vertices and a set* $\mathscr{S}$ *of nonempty subsets of* V *called* **simplices** *(singular:* **simplex***) such that the following hold.*

1. *Every singleton subset of* V *is a simplex.*
2. *Every nonempty subset of a simplex is a simplex.*

Definition. *A* **simplicial complex** *is a pair $K = (V, \mathscr{S})$ of a set V of vertices and a set $\mathscr{S}$ of nonempty subsets of V called* **simplices** *(singular:* **simplex***) such that the following hold.*

1. *Every singleton subset of V is a simplex.*
2. *Every nonempty subset of a simplex is a simplex.*

Definition.

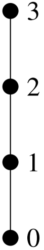
Definition. *A* **simplicial complex** *is a pair* $K = (V, \mathscr{S})$ *of a set* V *of vertices and a set* $\mathscr{S}$ *of nonempty subsets of* V *called* **simplices** *(singular:* **simplex***) such that the following hold.*

1. *Every singleton subset of* V *is a simplex.*
2. *Every nonempty subset of a simplex is a simplex.*

Definition. *For an ordered set* P*, the complex with vertex set* P *and set of simplices* $\mathscr{S}$ *being the set of totally ordered subsets is called the* **chain complex** *of* P.

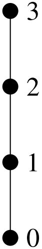
The Chain Complex

The Chain Complex



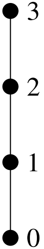
The Chain Complex

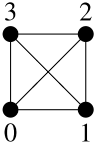
4



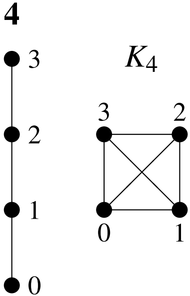
The Chain Complex

4



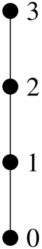


The Chain Complex

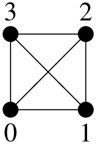


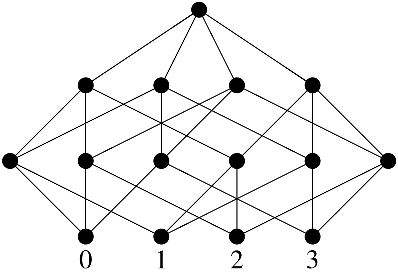
The Chain Complex

4



K_4

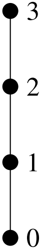




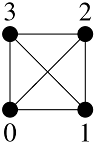
The Chain Complex

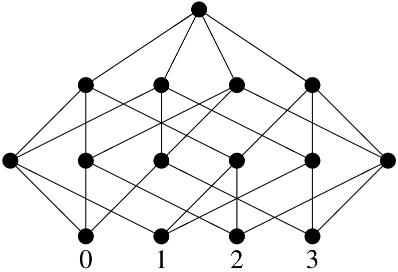
$$TCL(K_4) = TFL(\mathbf{4})$$

4



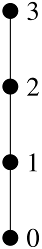
K_4



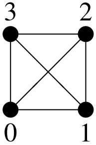


The Chain Complex

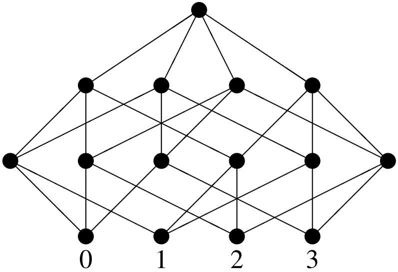
4

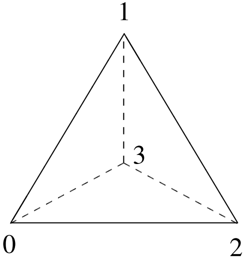


K_4



$$TCL(K_4) = TFL(\mathbf{4})$$





Definition.

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function.*

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function. Then* f *is called a* **simplicial map** *iff, for all* $S \in \mathscr{S}$ *we have* $f[S] \in \mathscr{T}$.

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function. Then* f *is called a* **simplicial map** *iff, for all* $S \in \mathscr{S}$ *we have* $f[S] \in \mathscr{T}$.

Proposition.

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function. Then* f *is called a* **simplicial map** *iff, for all* $S \in \mathscr{S}$ *we have* $f[S] \in \mathscr{T}$.

Proposition. *If* $f : P \to Q$ *is an order-preserving map*

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function. Then* f *is called a* **simplicial map** *iff, for all* $S \in \mathscr{S}$ *we have* $f[S] \in \mathscr{T}$.

Proposition. *If* $f : P \to Q$ *is an order-preserving map, then* f *is a simplicial map from the chain complex of* P *to the chain complex of* Q.

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function. Then* f *is called a* **simplicial map** *iff, for all* $S \in \mathscr{S}$ *we have* $f[S] \in \mathscr{T}$.

Proposition. *If* $f : P \to Q$ *is an order-preserving map, then* f *is a simplicial map from the chain complex of* P *to the chain complex of* Q.

Proof.

Definition. *Let* $K = (V, \mathscr{S})$ *and* $H = (W, \mathscr{T})$ *be simplicial complexes and let* $f : V \to W$ *be a function. Then* f *is called a* **simplicial map** *iff, for all* $S \in \mathscr{S}$ *we have* $f[S] \in \mathscr{T}$*.*

Proposition. *If* $f : P \to Q$ *is an order-preserving map, then* f *is a simplicial map from the chain complex of* P *to the chain complex of* Q*.*

Proof. Order-preserving maps map chains to chains.

Definition. *Let $K = (V, \mathscr{S})$ and $H = (W, \mathscr{T})$ be simplicial complexes and let $f : V \to W$ be a function. Then f is called a* **simplicial map** *iff, for all $S \in \mathscr{S}$ we have $f[S] \in \mathscr{T}$.*

Proposition. *If $f : P \to Q$ is an order-preserving map, then f is a simplicial map from the chain complex of P to the chain complex of Q.*

Proof. Order-preserving maps map chains to chains. ■

Definition.

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property**

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Proposition.

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Proposition. *Let* P *be an ordered set.*

Definition. *A simplicial complex $K = (V, \mathscr{S})$ has the* **fixed simplex property** *iff, for each simplicial map $f : V \to V$, there is a simplex S with $f[S] = S$.*

Proposition. *Let P be an ordered set. If the chain complex has the fixed simplex property*

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Proposition. *Let* P *be an ordered set. If the chain complex has the fixed simplex property, then* P *has the fixed point property*

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Proposition. *Let* P *be an ordered set. If the chain complex has the fixed simplex property, then* P *has the fixed point property, but not conversely.*

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Proposition. *Let* P *be an ordered set. If the chain complex has the fixed simplex property, then* P *has the fixed point property, but not conversely.*

Proof.

Definition. *A simplicial complex $K = (V, \mathscr{S})$ has the* **fixed simplex property** *iff, for each simplicial map $f : V \to V$, there is a simplex S with $f[S] = S$.*

Proposition. *Let P be an ordered set. If the chain complex has the fixed simplex property, then P has the fixed point property, but not conversely.*

Proof. Let $f : P \to P$ be order-preserving.

Definition. *A simplicial complex $K = (V, \mathscr{S})$ has the* **fixed simplex property** *iff, for each simplicial map $f : V \to V$, there is a simplex S with $f[S] = S$.*

Proposition. *Let P be an ordered set. If the chain complex has the fixed simplex property, then P has the fixed point property, but not conversely.*

Proof. Let $f : P \to P$ be order-preserving. Then f is a simplicial map on the chain complex.

Definition. *A simplicial complex $K = (V, \mathscr{S})$ has the* **fixed simplex property** *iff, for each simplicial map $f : V \to V$, there is a simplex S with $f[S] = S$.*

Proposition. *Let P be an ordered set. If the chain complex has the fixed simplex property, then P has the fixed point property, but not conversely.*

Proof. Let $f : P \to P$ be order-preserving. Then f is a simplicial map on the chain complex. Hence it fixes a simplex

Definition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the* **fixed simplex property** *iff, for each simplicial map* $f : V \to V$*, there is a simplex* S *with* $f[S] = S$*.*

Proposition. *Let* P *be an ordered set. If the chain complex has the fixed simplex property, then* P *has the fixed point property, but not conversely.*

Proof. Let $f : P \to P$ be order-preserving. Then f is a simplicial map on the chain complex. Hence it fixes a simplex, that is, f maps a chain to itself.

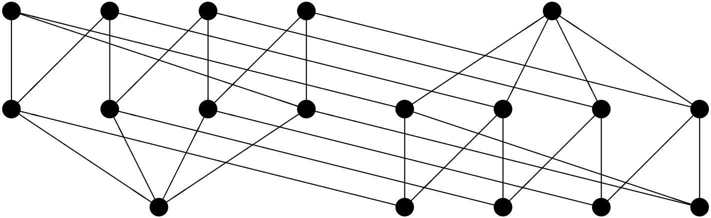
Definition. *A simplicial complex $K = (V, \mathscr{S})$ has the* **fixed simplex property** *iff, for each simplicial map $f : V \to V$, there is a simplex S with $f[S] = S$.*

Proposition. *Let P be an ordered set. If the chain complex has the fixed simplex property, then P has the fixed point property, but not conversely.*

Proof. Let $f : P \to P$ be order-preserving. Then f is a simplicial map on the chain complex. Hence it fixes a simplex, that is, f maps a chain to itself. Therefore, f must have a fixed point.

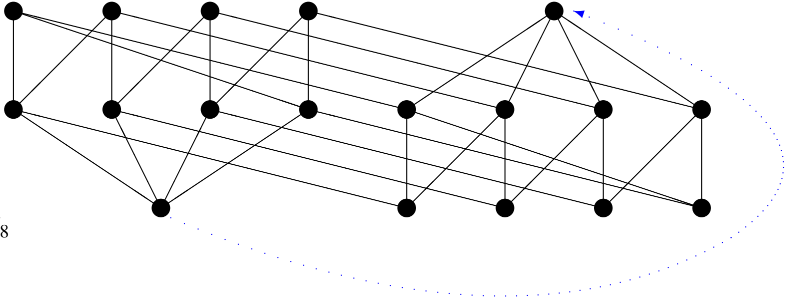
FPP for $P \not\Rightarrow$ FSP for Chain Complex

FPP for $P \not\Rightarrow$ FSP for Chain Complex



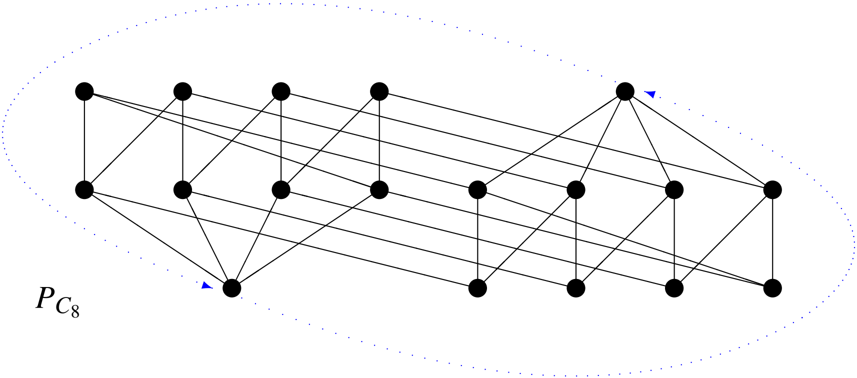
P_{C_8}

FPP for $P \not\Rightarrow$ FSP for Chain Complex

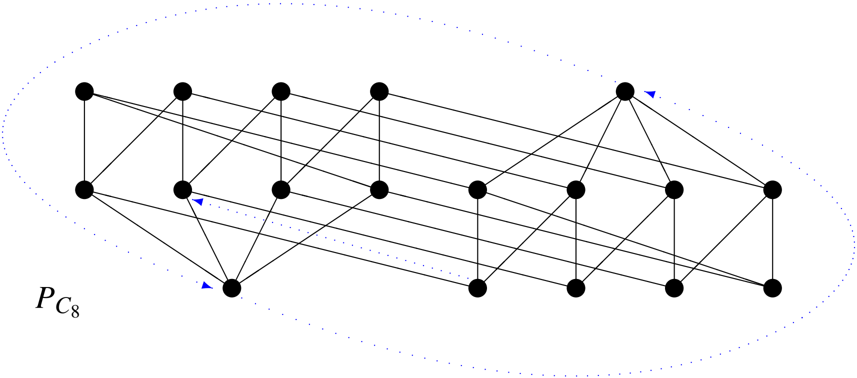


P_{C_8}

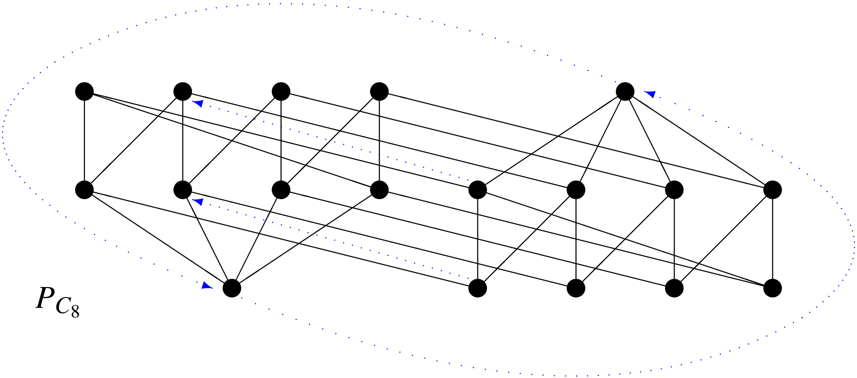
FPP for $P \not\Rightarrow$ FSP for Chain Complex



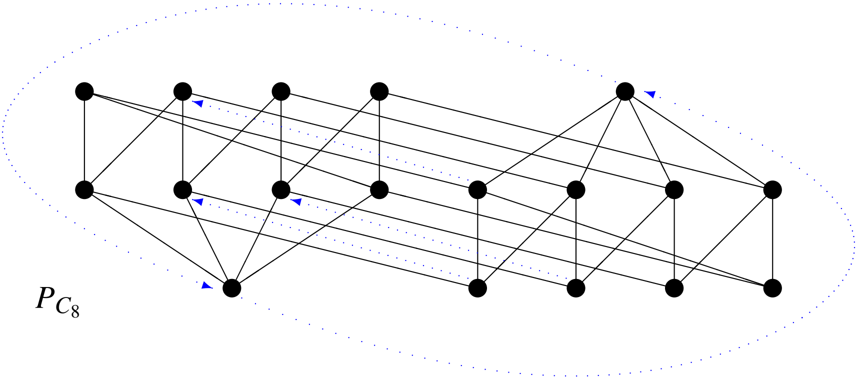
FPP for $P \not\Rightarrow$ FSP for Chain Complex



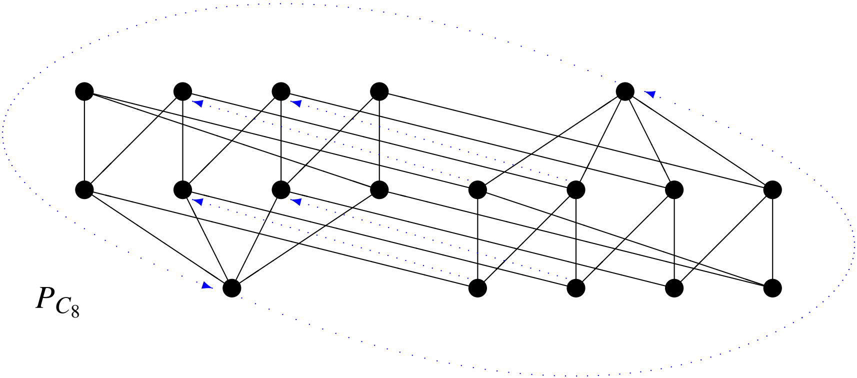
FPP for $P \not\Rightarrow$ FSP for Chain Complex



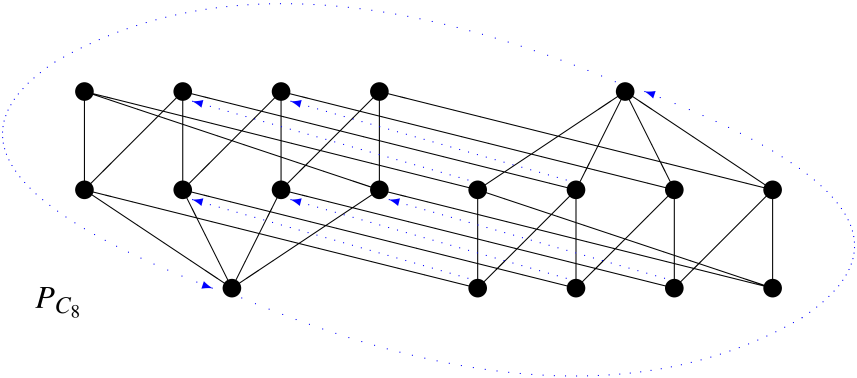
FPP for $P \not\Rightarrow$ FSP for Chain Complex



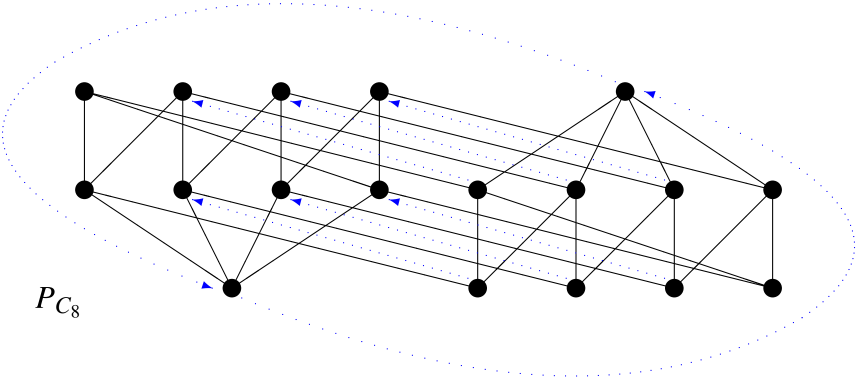
FPP for $P \not\Rightarrow$ FSP for Chain Complex



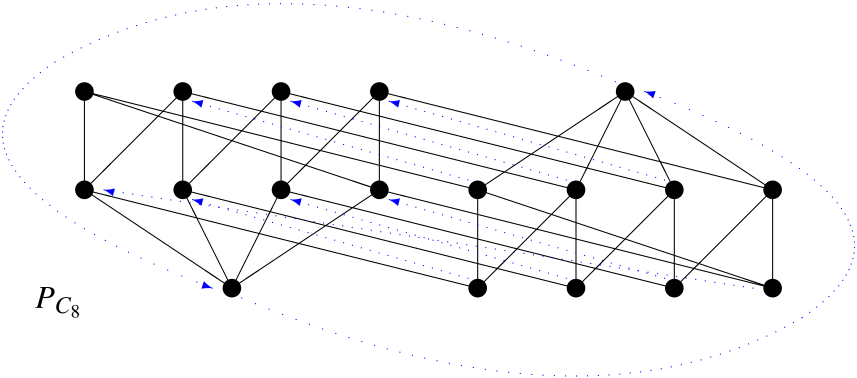
FPP for $P \not\Rightarrow$ FSP for Chain Complex



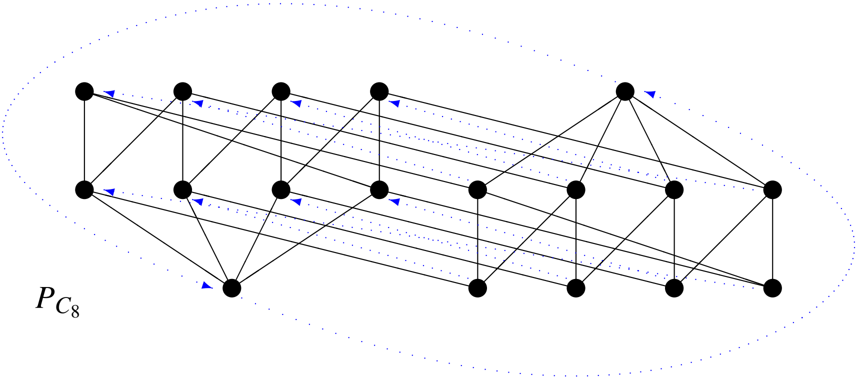
FPP for $P \not\Rightarrow$ FSP for Chain Complex



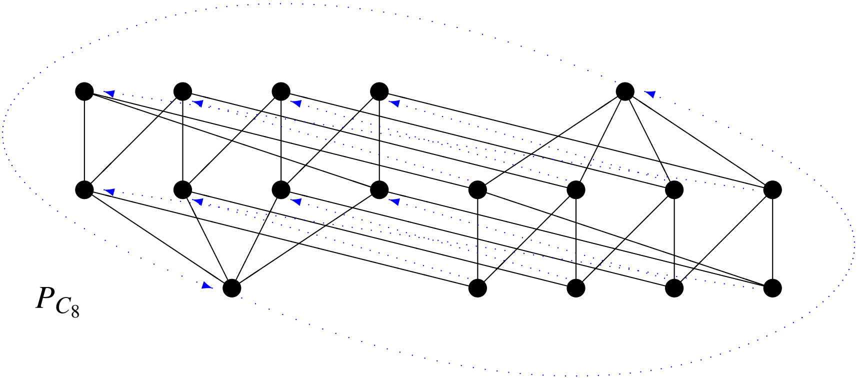
FPP for $P \not\Rightarrow$ FSP for Chain Complex



FPP for $P \not\Rightarrow$ FSP for Chain Complex

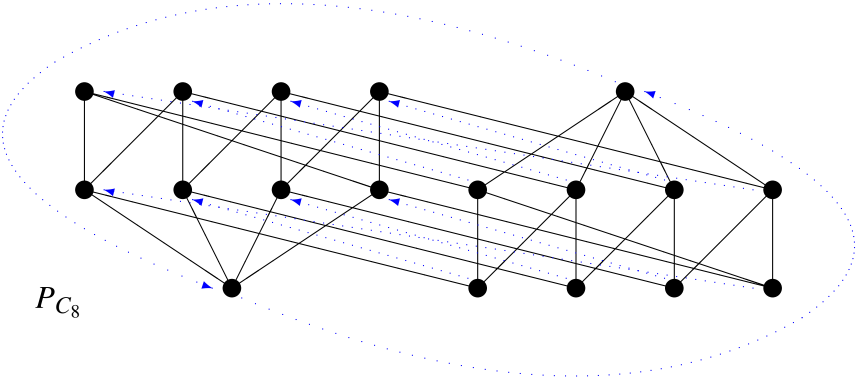


FPP for $P \not\Rightarrow$ FSP for Chain Complex



"and vice versa for the upper crown."

FPP for $P \not\Rightarrow$ FSP for Chain Complex



"and vice versa for the upper crown." ■

Proposition.

Proposition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the fixed simplex property iff* $\mathscr{S}$*, ordered by inclusion, has the fixed point property.*

Proposition. *A simplicial complex $K = (V, \mathscr{S})$ has the fixed simplex property iff $\mathscr{S}$, ordered by inclusion, has the fixed point property.*

Proof.

Proposition. *A simplicial complex $K = (V, \mathscr{S})$ has the fixed simplex property iff $\mathscr{S}$, ordered by inclusion, has the fixed point property.*

Proof. For "$\Leftarrow$," note that every simplicial map induces an order-preserving map on $\mathscr{S}$.

Proposition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the fixed simplex property iff* $\mathscr{S}$*, ordered by inclusion, has the fixed point property.*

Proof. For "$\Leftarrow$," note that every simplicial map induces an order-preserving map on $\mathscr{S}$.
For "$\Rightarrow$," note that every order-preserving map $f : \mathscr{S} \to \mathscr{S}$

Proposition. *A simplicial complex* $K = (V, \mathscr{S})$ *has the fixed simplex property iff* $\mathscr{S}$*, ordered by inclusion, has the fixed point property.*

Proof. For "$\Leftarrow$," note that every simplicial map induces an order-preserving map on $\mathscr{S}$.

For "$\Rightarrow$," note that every order-preserving map $f : \mathscr{S} \to \mathscr{S}$ is above an order-preserving map f_* that maps minimal elements to minimal elements

Proposition. *A simplicial complex $K = (V, \mathscr{S})$ has the fixed simplex property iff $\mathscr{S}$, ordered by inclusion, has the fixed point property.*

Proof. For "$\Leftarrow$," note that every simplicial map induces an order-preserving map on $\mathscr{S}$.
For "$\Rightarrow$," note that every order-preserving map $f : \mathscr{S} \to \mathscr{S}$ is above an order-preserving map f_* that maps minimal elements to minimal elements and that f_* is above the function that maps each $\sigma \in \mathscr{S}$ to $\{f_*(x) : x \in \sigma\}$

Proposition. *A simplicial complex $K = (V, \mathscr{S})$ has the fixed simplex property iff $\mathscr{S}$, ordered by inclusion, has the fixed point property.*

Proof. For "$\Leftarrow$," note that every simplicial map induces an order-preserving map on $\mathscr{S}$.
For "$\Rightarrow$," note that every order-preserving map $f : \mathscr{S} \to \mathscr{S}$ is above an order-preserving map f_* that maps minimal elements to minimal elements and that f_* is above the function that maps each $\sigma \in \mathscr{S}$ to $\{f_*(x) : x \in \sigma\}$, which is induced by a simplicial map.

Proposition. *A simplicial complex $K = (V, \mathscr{S})$ has the fixed simplex property iff $\mathscr{S}$, ordered by inclusion, has the fixed point property.*

Proof. For "$\Leftarrow$," note that every simplicial map induces an order-preserving map on $\mathscr{S}$.
For "$\Rightarrow$," note that every order-preserving map $f : \mathscr{S} \to \mathscr{S}$ is above an order-preserving map f_* that maps minimal elements to minimal elements and that f_* is above the function that maps each $\sigma \in \mathscr{S}$ to $\{f_*(x) : x \in \sigma\}$, which is induced by a simplicial map. ■

Definition.

Definition. *Let* $K = (V, \mathscr{S})$ *be a simplicial complex.*

Definition. *Let* $K = (V, \mathscr{S})$ *be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of* K

Definition. *Let* $K=(V,\mathscr{S})$ *be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of* K *is a set of simplexes in* $\mathbb{R}^d$

Definition. *Let* $K = (V, \mathscr{S})$ *be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of* K *is a set of simplexes in* $\mathbb{R}^d$ *so that the vertices of* K *are identified with points in* $\mathbb{R}^d$

Definition. *Let* $K=(V,\mathscr{S})$ *be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of* K *is a set of simplexes in* $\mathbb{R}^d$ *so that the vertices of* K *are identified with points in* $\mathbb{R}^d$ *so that all convex hulls of images of sets in* $\mathscr{S}$ *are nontrivial*

Definition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of K is a set of simplexes in $\mathbb{R}^d$ so that the vertices of K are identified with points in $\mathbb{R}^d$ so that all convex hulls of images of sets in $\mathscr{S}$ are nontrivial and so that the simplices in $\mathscr{R}(K)$ are exactly the convex hulls of images of the sets in $\mathscr{S}$.*

Definition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of K is a set of simplexes in $\mathbb{R}^d$ so that the vertices of K are identified with points in $\mathbb{R}^d$ so that all convex hulls of images of sets in $\mathscr{S}$ are nontrivial and so that the simplices in $\mathscr{R}(K)$ are exactly the convex hulls of images of the sets in $\mathscr{S}$.*

Proposition.

Definition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. A* **topological realization** *$\mathscr{R}(K)$ of K is a set of simplexes in $\mathbb{R}^d$ so that the vertices of K are identified with points in $\mathbb{R}^d$ so that all convex hulls of images of sets in $\mathscr{S}$ are nontrivial and so that the simplices in $\mathscr{R}(K)$ are exactly the convex hulls of images of the sets in $\mathscr{S}$.*

Proposition. *Let $K = (V, \mathscr{S})$ and $H = (W, \mathscr{T})$ be simplicial complexes and let $f : V \to W$ be a simplicial map.*

Definition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. A* **topological realization** $\mathscr{R}(K)$ *of K is a set of simplexes in $\mathbb{R}^d$ so that the vertices of K are identified with points in $\mathbb{R}^d$ so that all convex hulls of images of sets in $\mathscr{S}$ are nontrivial and so that the simplices in $\mathscr{R}(K)$ are exactly the convex hulls of images of the sets in $\mathscr{S}$.*

Proposition. *Let $K = (V, \mathscr{S})$ and $H = (W, \mathscr{T})$ be simplicial complexes and let $f : V \to W$ be a simplicial map. Then f can be extended to an affine map from $\mathscr{R}(K)$ to $\mathscr{R}(H)$.*

Proposition.

Proposition. *Let $K = (V, \mathscr{S})$ be a simplicial complex.*

Proposition. *Let* $K = (V, \mathscr{S})$ *be a simplicial complex. If* $\mathscr{R}(K)$ *has the topological fixed point property*

Proposition. *Let $K=(V,\mathscr{S})$ be a simplicial complex. If $\mathscr{R}(K)$ has the topological fixed point property, then K has the fixed simplex property, but not conversely.*

Proposition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. If $\mathscr{R}(K)$ has the topological fixed point property, then K has the fixed simplex property, but not conversely.*

Proof.

Proposition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. If $\mathscr{R}(K)$ has the topological fixed point property, then K has the fixed simplex property, but not conversely.*

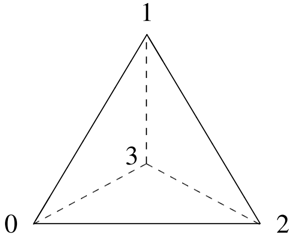
Proof. Every simplicial map induces an affine map

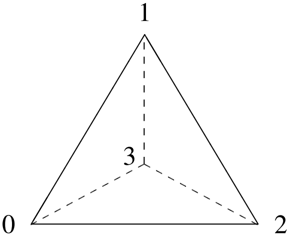
Proposition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. If $\mathscr{R}(K)$ has the topological fixed point property, then K has the fixed simplex property, but not conversely.*

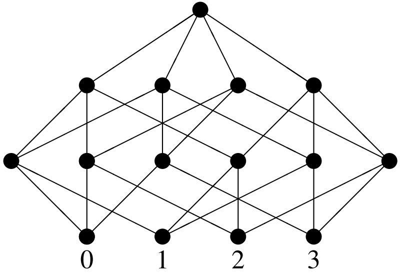
Proof. Every simplicial map induces an affine map, which is continuous and hence has a fixed point.

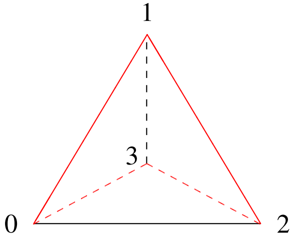
Proposition. *Let $K = (V, \mathscr{S})$ be a simplicial complex. If $\mathscr{R}(K)$ has the topological fixed point property, then K has the fixed simplex property, but not conversely.*

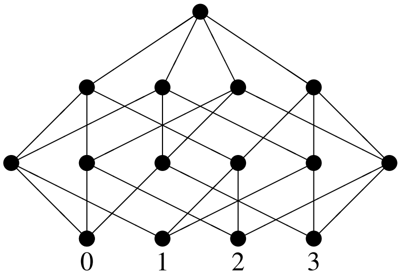
Proof. Every simplicial map induces an affine map, which is continuous and hence has a fixed point.
The affine map can only have a fixed point if it maps a simplex to itself.

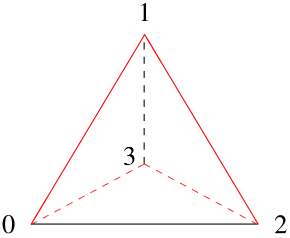

1
3
0
2

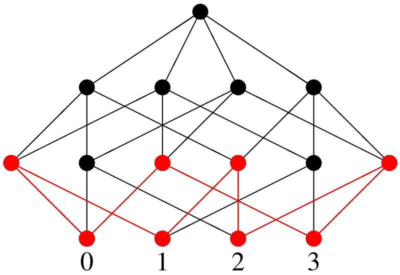

1
3
0
2

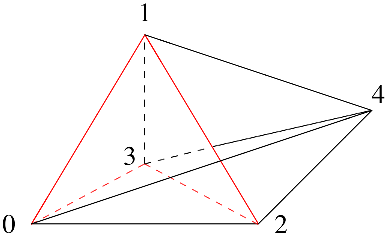

0
1
2
3

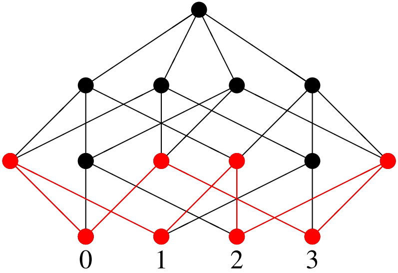

1
3
0
2

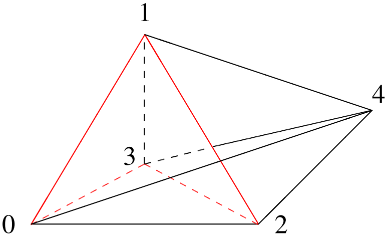

0
1
2
3


1
3
0
2

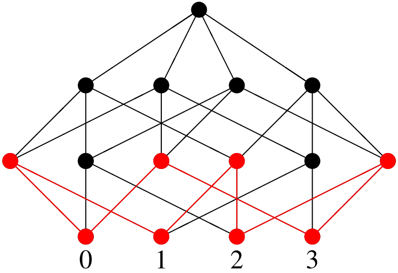

0
1
2
3

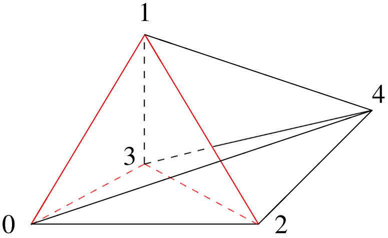

1
4
3
0
2


0 1 2 3

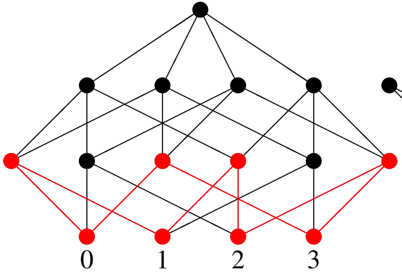


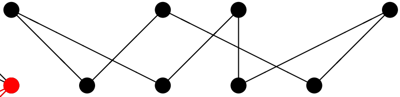
Retracts to S^1.

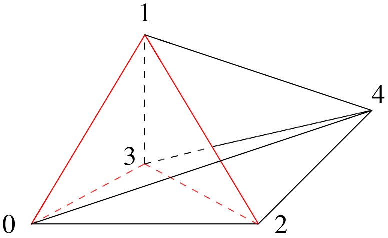




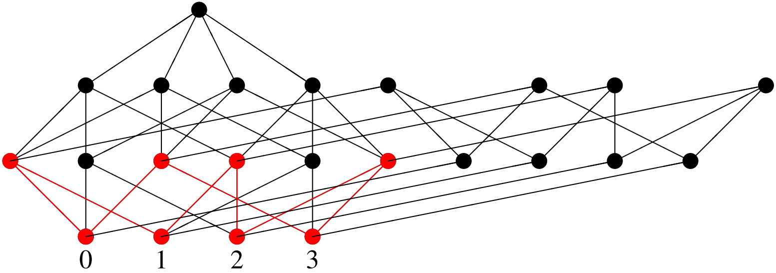
Retracts to S^1.

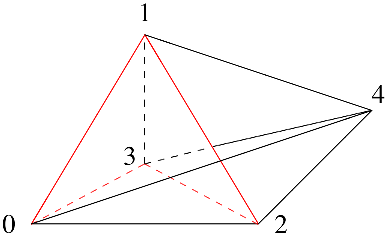




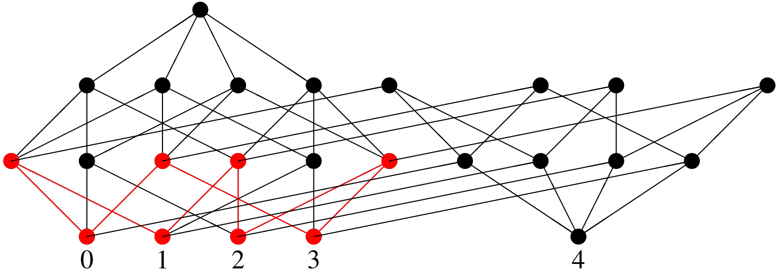


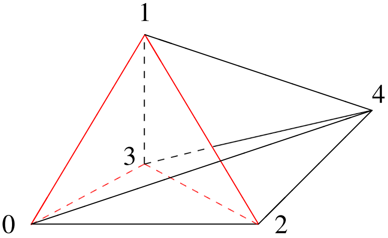
Retracts to S^1.



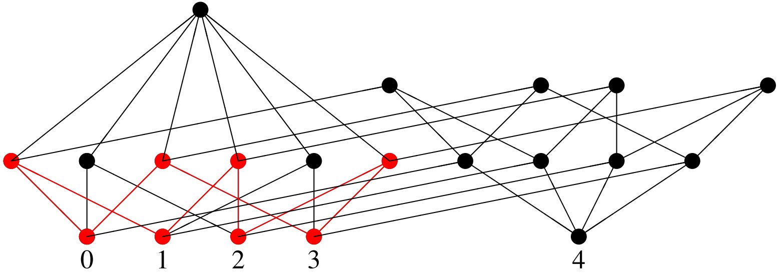


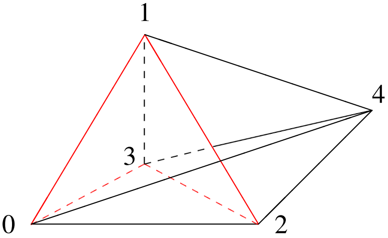
Retracts to S^1.



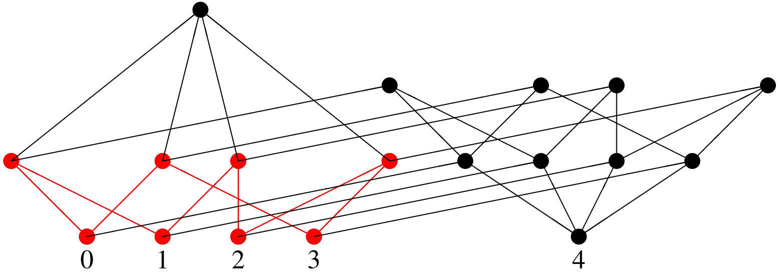


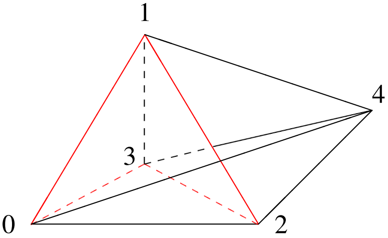
Retracts to S^1.



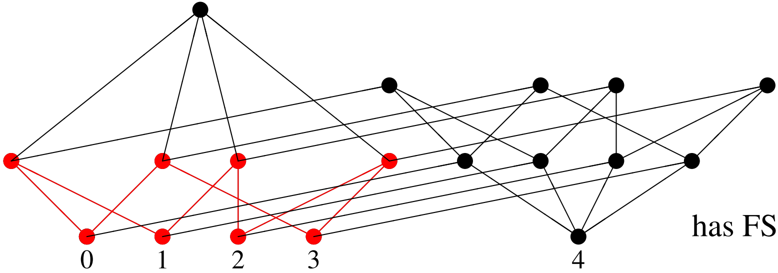


Retracts to S^1.





Retracts to S^1.



has FSP

Let $K = (V, \mathscr{S})$ be a simplicial complex.

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α.

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex

Let $K = (V, \mathscr{S})$ be a simplicial complex.

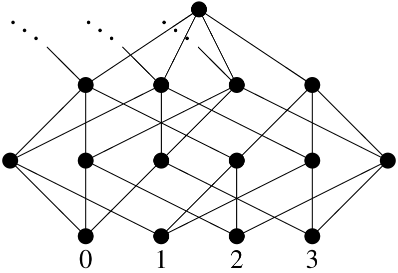
1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise.

Let $K = (V, \mathscr{S})$ be a simplicial complex.

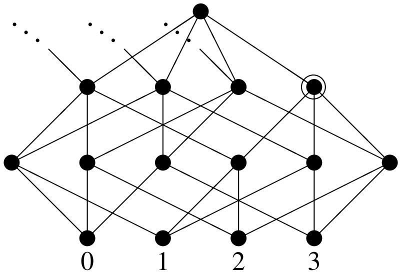
1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise. In this case, we also write $K(e) := L$.

Visualizing an Elementary Collapse

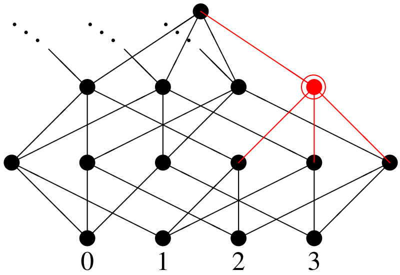
Visualizing an Elementary Collapse



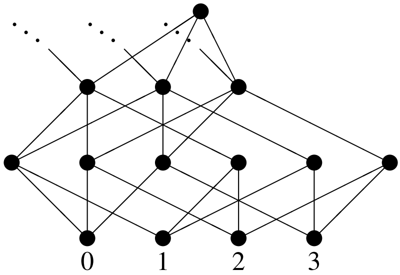
Visualizing an Elementary Collapse



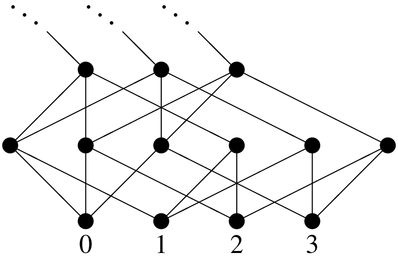
Visualizing an Elementary Collapse



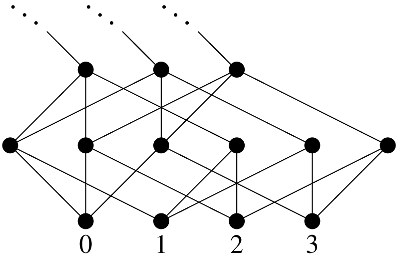
Visualizing an Elementary Collapse



Visualizing an Elementary Collapse



Visualizing an Elementary Collapse



!!!

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise. In this case, we also write $K(e) := L$.

Let $K=(V,\mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha,\beta\}\subseteq\mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e=\{\alpha,\beta\}$ so that $L=(V',\mathscr{S}\setminus\{\alpha,\beta\})$, where $V'=V$ if α has more than one vertex and $V'=V\setminus\alpha$ otherwise. In this case, we also write $K(e):=L$.
3. A **sequence of elementary collapses** in K is a sequence $\mathbf{e}=(e_1,\ldots,e_n)$ so that

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise. In this case, we also write $K(e) := L$.
3. A **sequence of elementary collapses** in K is a sequence $\mathbf{e} = (e_1, \ldots, e_n)$ so that e_1 is an end in $K_1 := K$

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise. In this case, we also write $K(e) := L$.
3. A **sequence of elementary collapses** in K is a sequence $\mathbf{e} = (e_1, \ldots, e_n)$ so that e_1 is an end in $K_1 := K$ and so that, for $i = 2, \ldots, n$, e_i is an end of $K_i := K_{i-1}(e_{i-1})$.

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise. In this case, we also write $K(e) := L$.
3. A **sequence of elementary collapses** in K is a sequence $\mathbf{e} = (e_1, \ldots, e_n)$ so that e_1 is an end in $K_1 := K$ and so that, for $i = 2, \ldots, n$, e_i is an end of $K_i := K_{i-1}(e_{i-1})$. In this case, we say that K collapses to $K(\mathbf{e}) = K_n(e_n)$.

Let $K = (V, \mathscr{S})$ be a simplicial complex.

1. An **end** is a pair $\{\alpha, \beta\} \subseteq \mathscr{S}$ of simplices so that β is the unique simplex that strictly contains α. The dimension of the end is the dimension d of β.
2. The subcomplex L of K is obtained from K by an **elementary collapse** iff there is an end $e = \{\alpha, \beta\}$ so that $L = (V', \mathscr{S} \setminus \{\alpha, \beta\})$, where $V' = V$ if α has more than one vertex and $V' = V \setminus \alpha$ otherwise. In this case, we also write $K(e) := L$.
3. A **sequence of elementary collapses** in K is a sequence $\mathbf{e} = (e_1, \ldots, e_n)$ so that e_1 is an end in $K_1 := K$ and so that, for $i = 2, \ldots, n$, e_i is an end of $K_i := K_{i-1}(e_{i-1})$. In this case, we say that K collapses to $K(\mathbf{e}) = K_n(e_n)$.
4. K is called **collapsible** (in Whitehead's sense) iff K collapses to one of its vertices.

Properties of Collapsible Complexes

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic.

Properties of Collapsible Complexes

1. If $K=(V,\mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property.

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof.

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)
3. There are collapsible simplicial complexes that can be collapsed onto non-collapsible subcomplexes.

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)
3. There are collapsible simplicial complexes that can be collapsed onto non-collapsible subcomplexes. (?!)

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)
3. There are collapsible simplicial complexes that can be collapsed onto non-collapsible subcomplexes. (?!)
4. Checking this type of collapsibility is polynomial for dimension ≤ 2, NP-complete for dimension ≥ 3.

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)
3. There are collapsible simplicial complexes that can be collapsed onto non-collapsible subcomplexes. (?!)
4. Checking this type of collapsibility is polynomial for dimension ≤ 2, NP-complete for dimension ≥ 3. (History seems complicated.

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)
3. There are collapsible simplicial complexes that can be collapsed onto non-collapsible subcomplexes. (?!)
4. Checking this type of collapsibility is polynomial for dimension ≤ 2, NP-complete for dimension ≥ 3. (History seems complicated. An argument by Haken in 1973 may not be completely correct

Properties of Collapsible Complexes

1. If $K = (V, \mathscr{S})$ is collapsible, then K is acyclic. Proof, obviously, requires algebraic topology.
2. If $K = (V, \mathscr{S})$ is collapsible, then K has the fixed simplex property. Follows from acyclicity, but recently Baclawski published a combinatorial proof. (Counting argument using contradiction.)
3. There are collapsible simplicial complexes that can be collapsed onto non-collapsible subcomplexes. (?!)
4. Checking this type of collapsibility is polynomial for dimension ≤ 2, NP-complete for dimension ≥ 3. (History seems complicated. An argument by Haken in 1973 may not be completely correct, a 2012 argument by Tancer is on the arXiV.)

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

1. Con: There will most likely not be a proof that says "It works for K iff it works for $K(e)$,"

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

1. Con: There will most likely not be a proof that says “It works for K iff it works for $K(e)$,” because of the earlier example.

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

1. Con: There will most likely not be a proof that says "It works for K iff it works for $K(e)$," because of the earlier example.
2. Pro: The argument for ordered sets of order dimension 2 is not this kind of "pointwise" argument.

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

1. Con: There will most likely not be a proof that says "It works for K iff it works for $K(e)$," because of the earlier example.
2. Pro: The argument for ordered sets of order dimension 2 is not this kind of "pointwise" argument.
3. Con: If you choose the wrong collapsing sequence, you might collapse onto something non-collapsible.

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

1. Con: There will most likely not be a proof that says "It works for K iff it works for $K(e)$," because of the earlier example.
2. Pro: The argument for ordered sets of order dimension 2 is not this kind of "pointwise" argument.
3. Con: If you choose the wrong collapsing sequence, you might collapse onto something non-collapsible.
4. Pro: The argument for ordered sets of order dimension 2 simply uses the idea that there is *one* collapsing sequence onto a singleton.

Does this type of collapsibility imply that there is a consistency enforcing algorithm for the network that encodes search for a fixed point free order-preserving map that returns an empty network?

1. Con: There will most likely not be a proof that says "It works for K iff it works for $K(e)$," because of the earlier example.
2. Pro: The argument for ordered sets of order dimension 2 is not this kind of "pointwise" argument.
3. Con: If you choose the wrong collapsing sequence, you might collapse onto something non-collapsible.
4. Pro: The argument for ordered sets of order dimension 2 simply uses the idea that there is *one* collapsing sequence onto a singleton. (Though, for order dimension 2, the order of removal does not matter.)

Questions

Questions

1. Is every *simplicial* retract of a collapsible simplicial complex collapsible?

Introduction

Introduction

1.

Introduction

Introduction

Text

Example.

Example.

Example.

Example.

Example.

Theorem.

Theorem.

Theorem.

Proof.

Theorem.

Proof.

Theorem.

Proof. ■

Example.

Example.

Example.

Example.

Example.

□