Matroids with many small circuits and many small cocircuits

James Oxley, Louisiana State University
Simon Pfeil (presenting), Louisiana State University
Charles Semple, University of Canterbury
Geoff Whittle, Victoria University of Wellington

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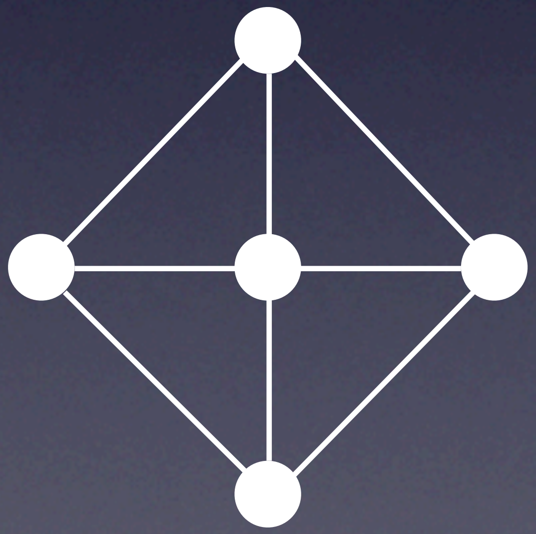

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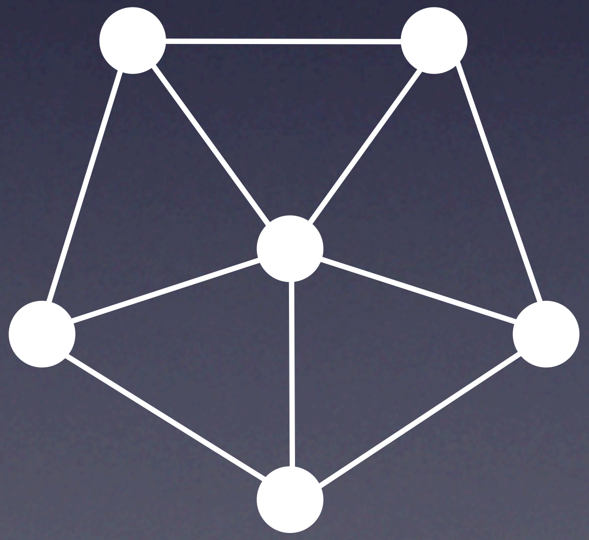

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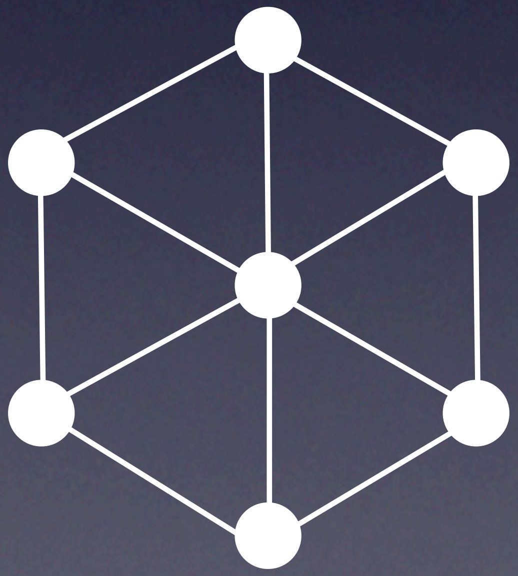

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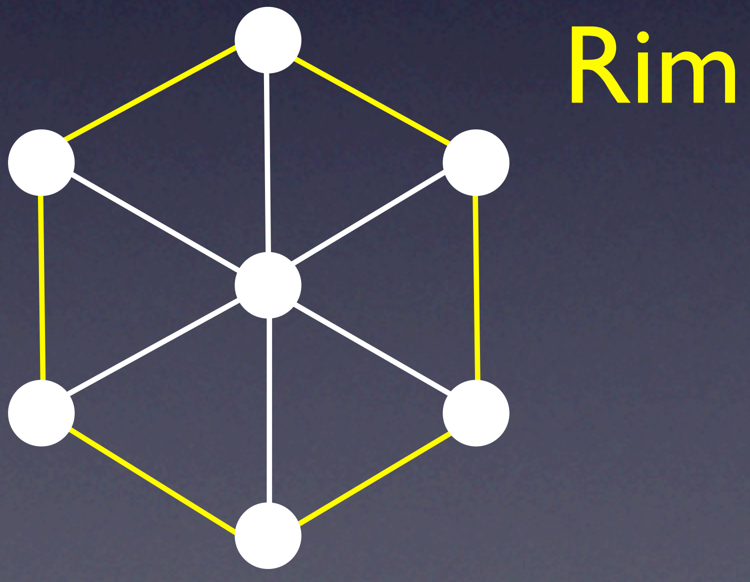

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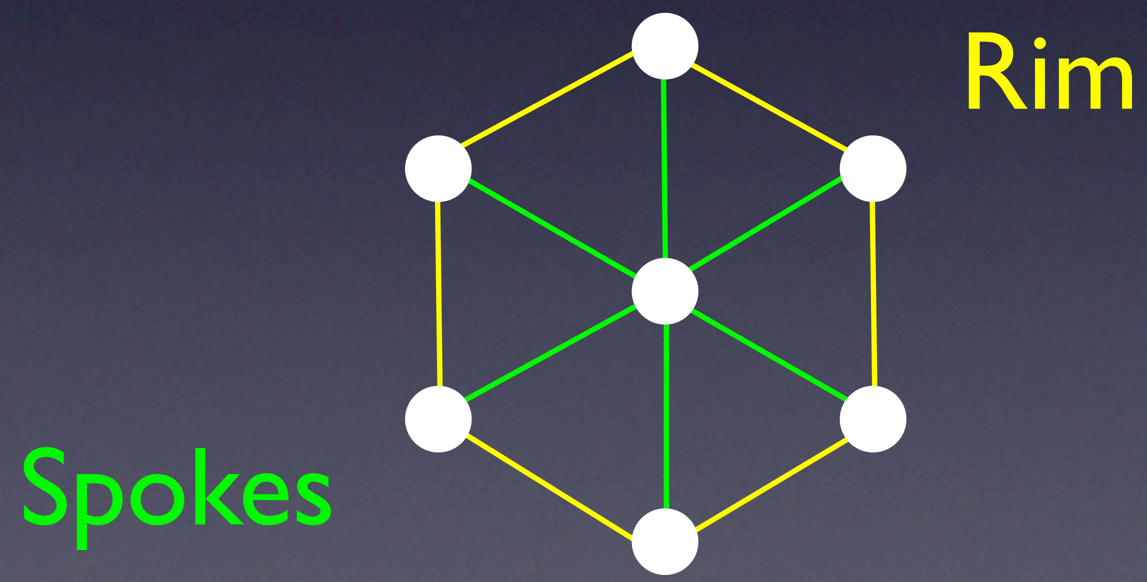

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From graphs to matroids:

edges $\leftrightarrow$ elements

cycles $\leftrightarrow$ circuits

More matroid stuff to know:

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Connectivity:

If a matroid is n-connected,
then its smallest (co)circuits have size n.

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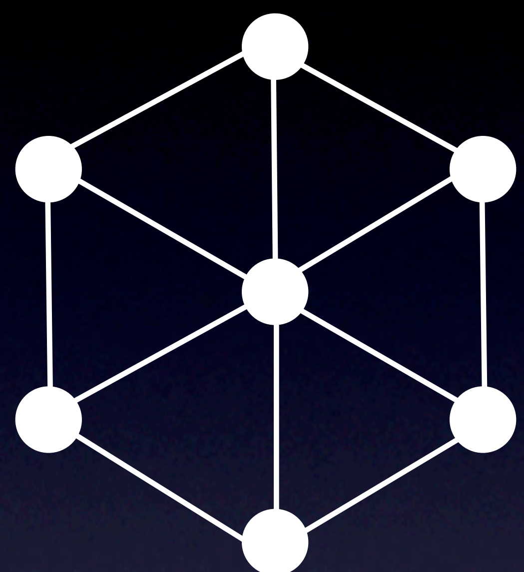

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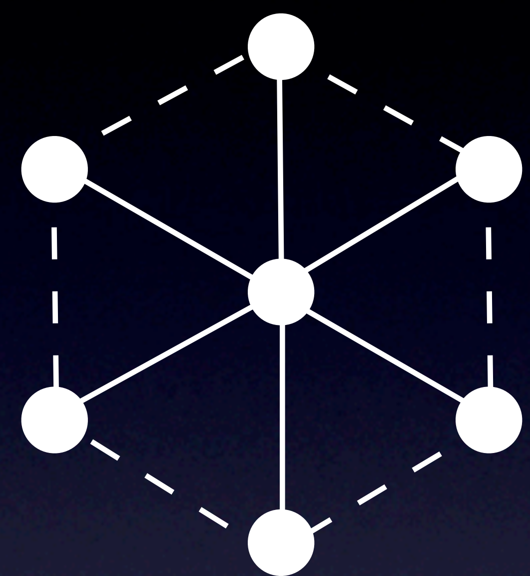

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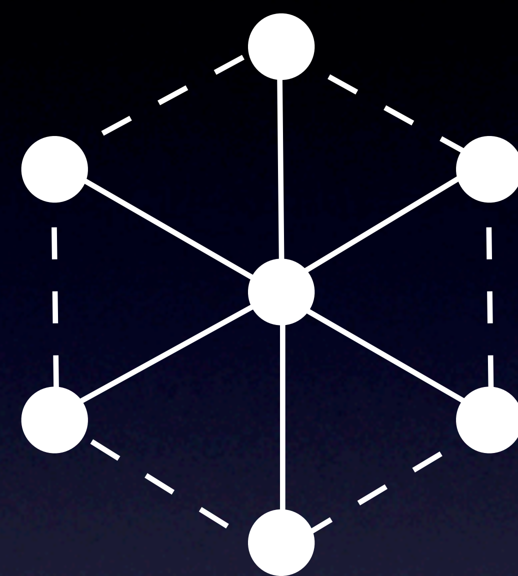

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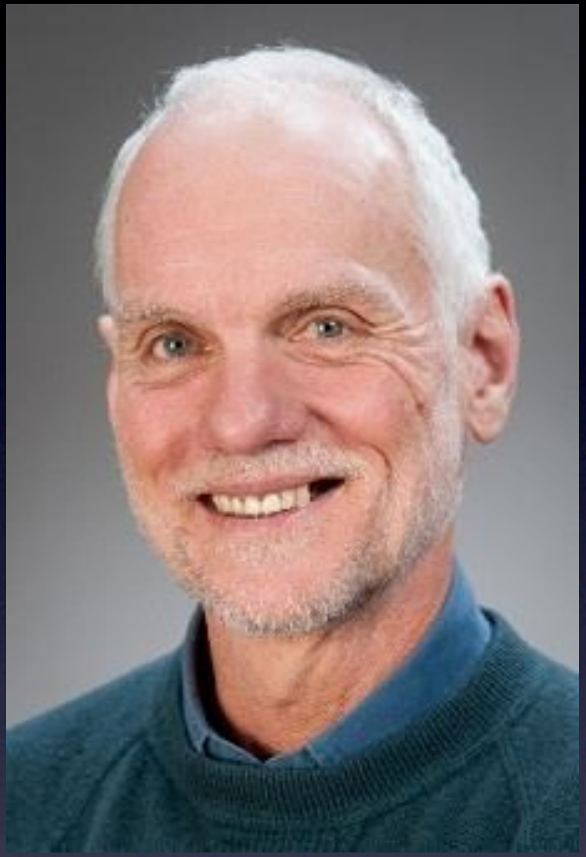

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The only 4-connected matroids in which every pair of elements is in a 4-element circuit and every element is in a 4-element cocircuit are $M(K_{4,n,})$ if $n \geq 4$, and $|M| \geq 16$!

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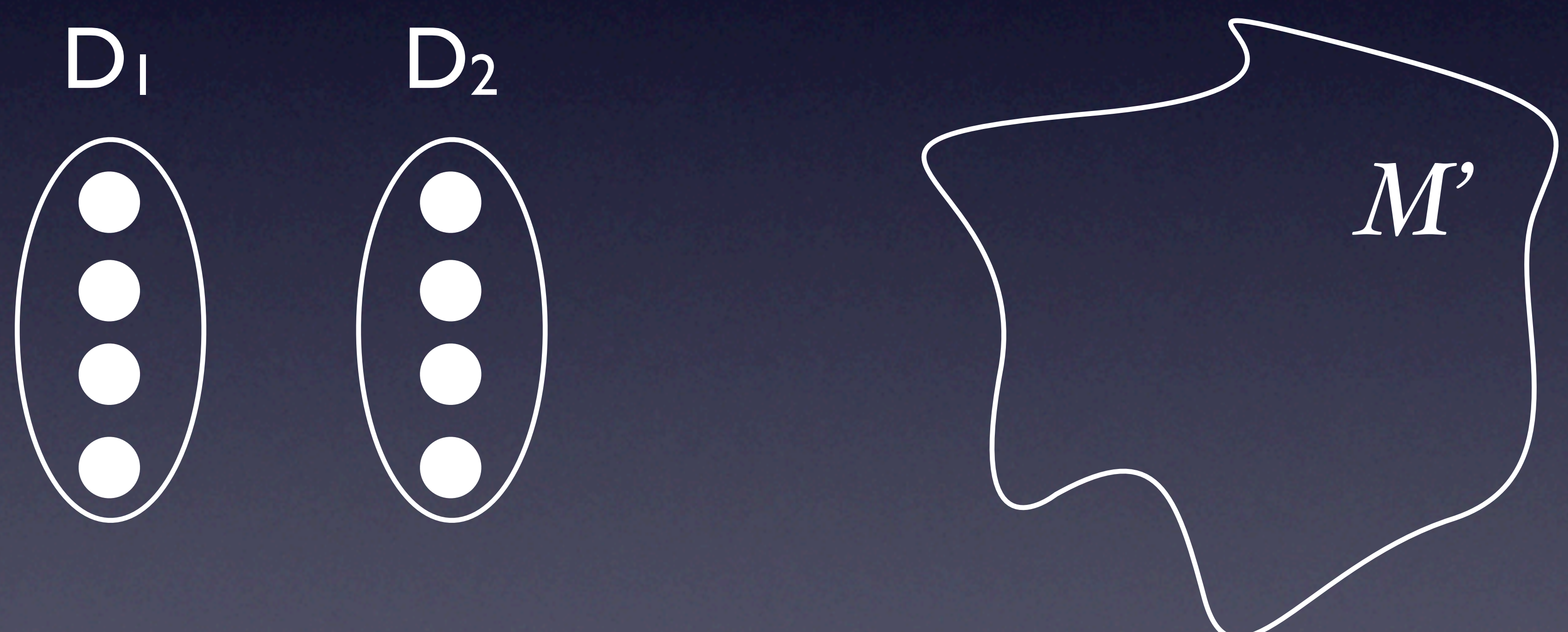

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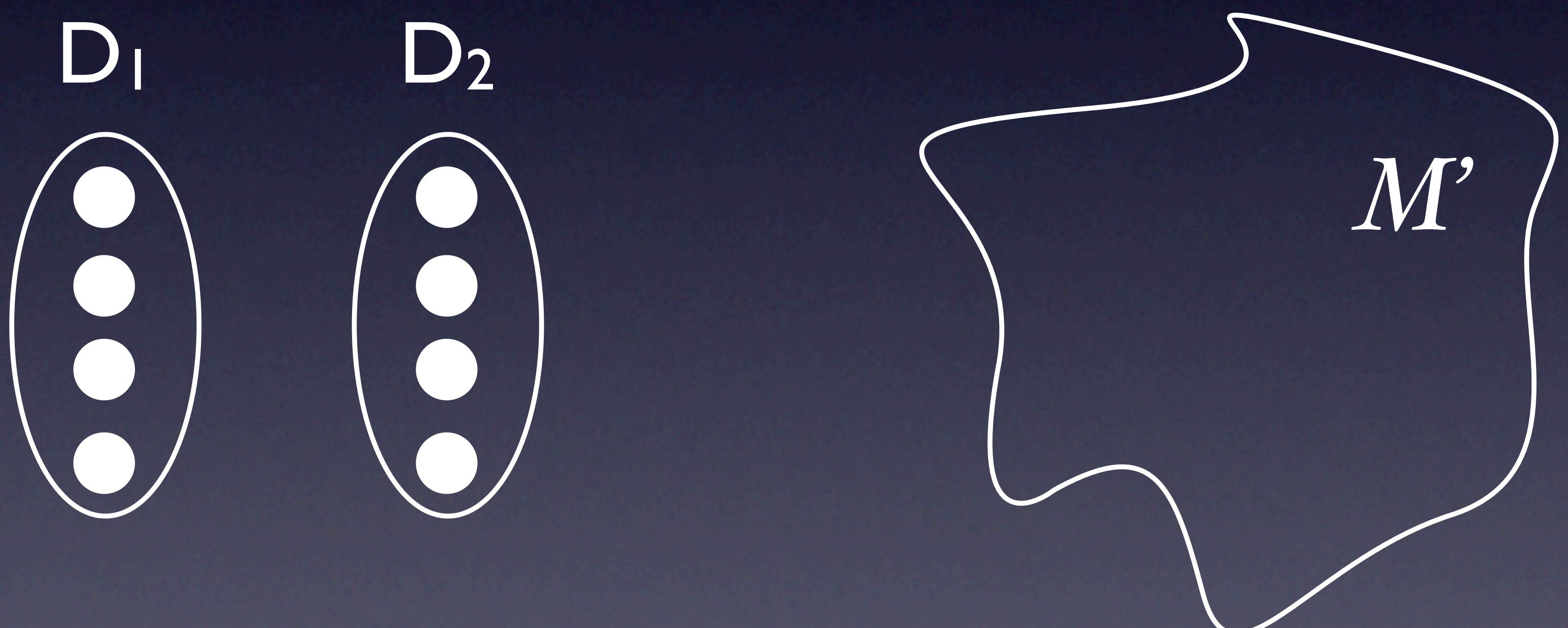

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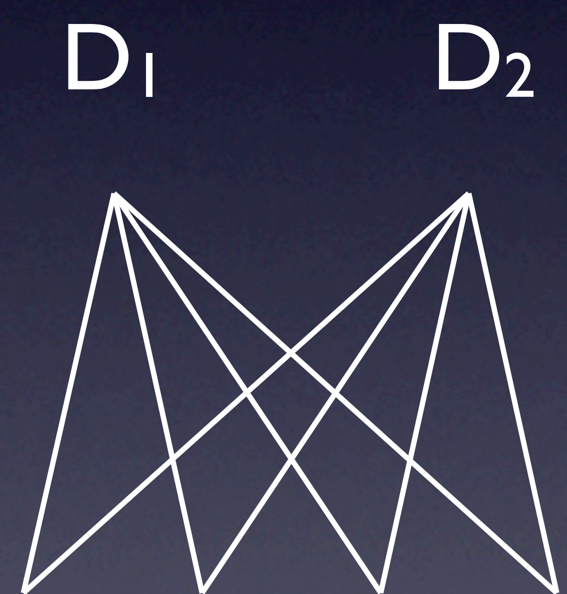

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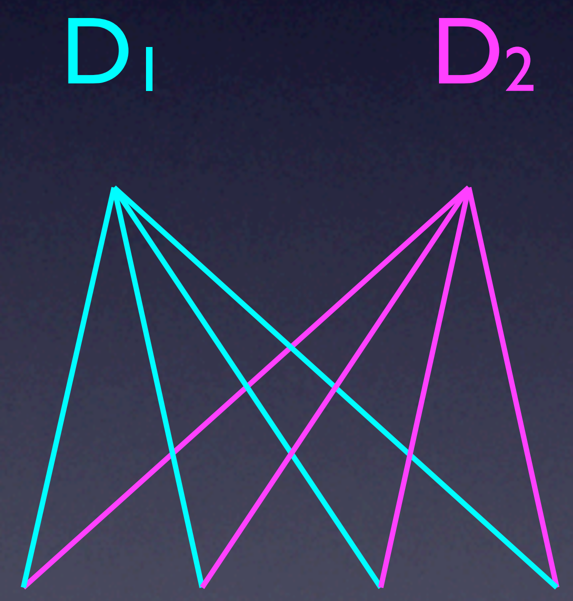

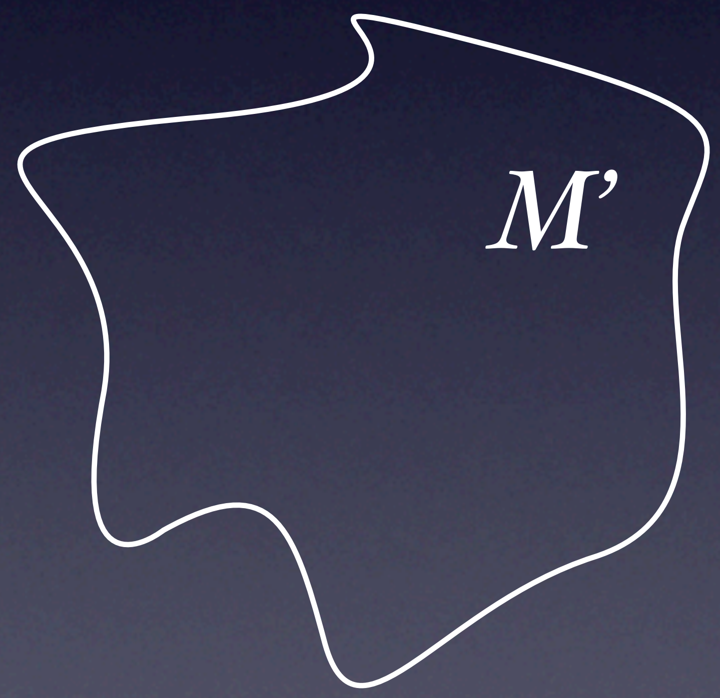

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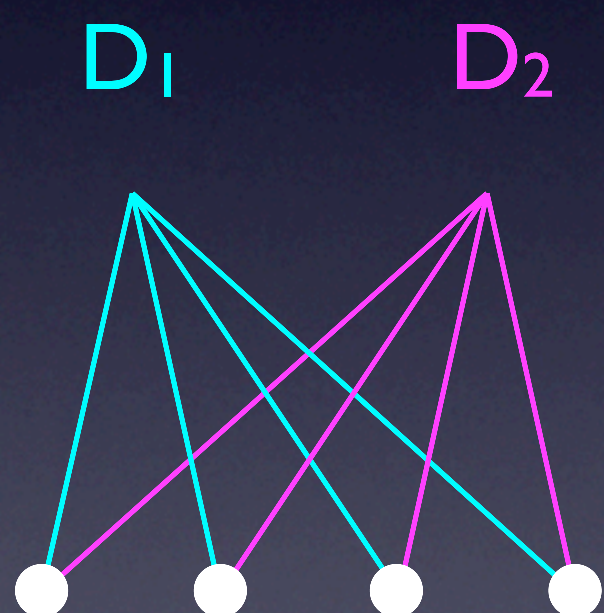

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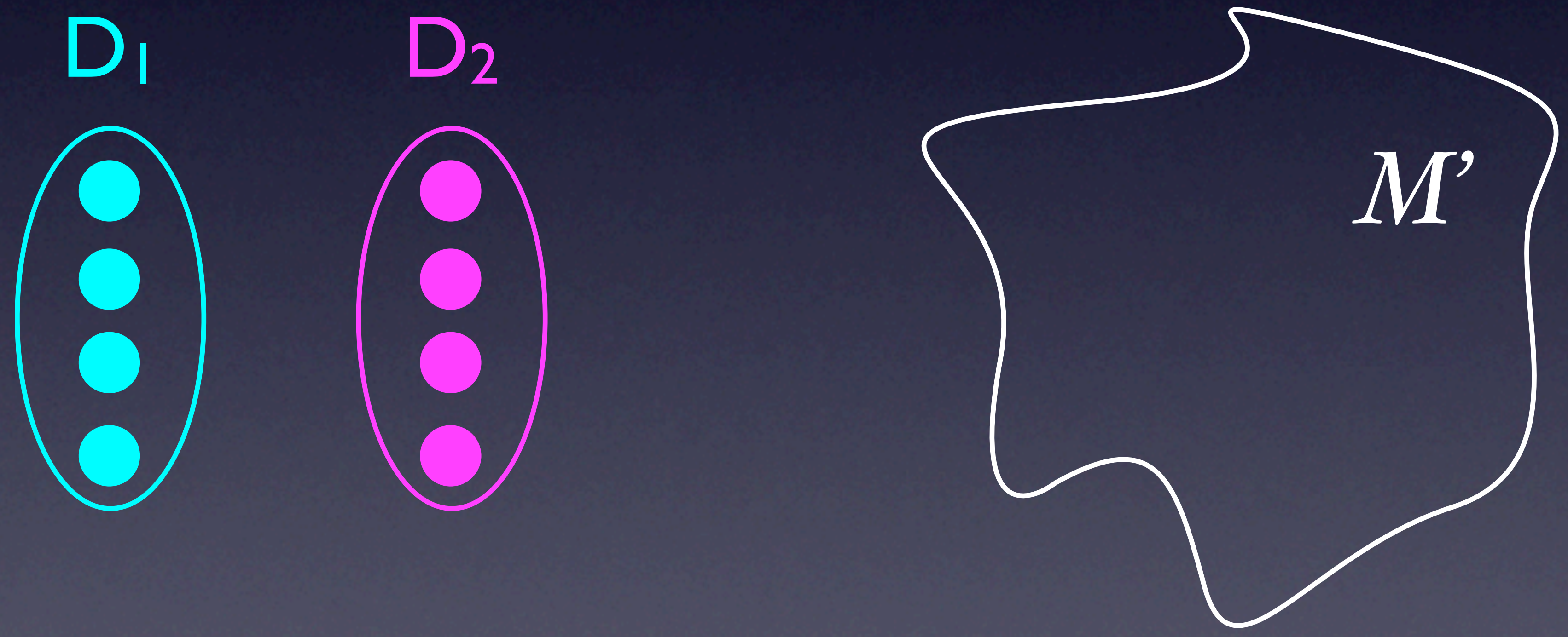

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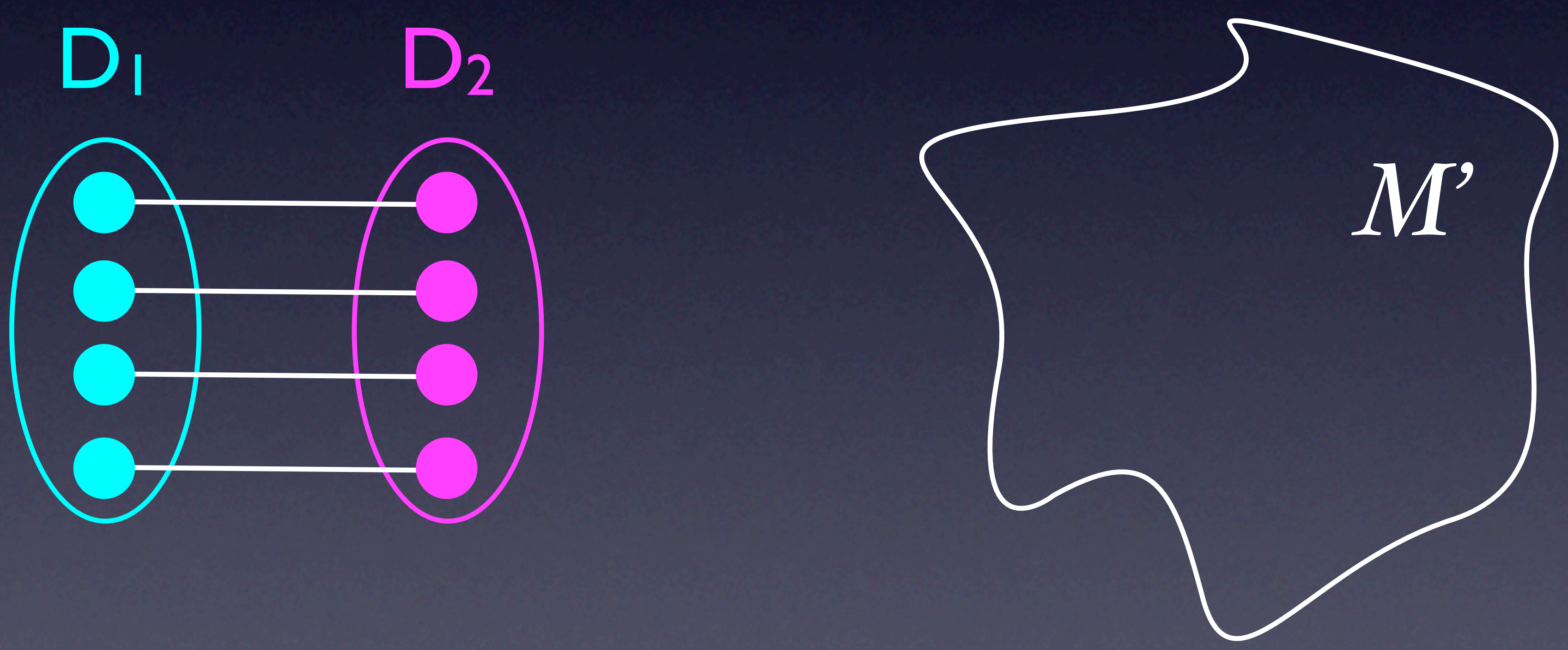

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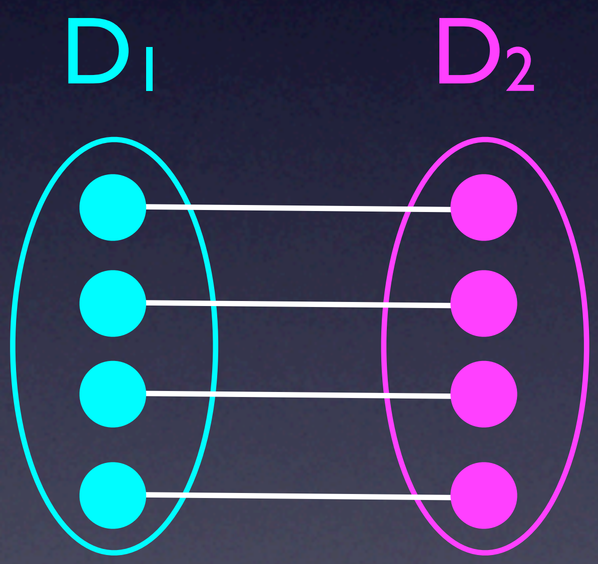

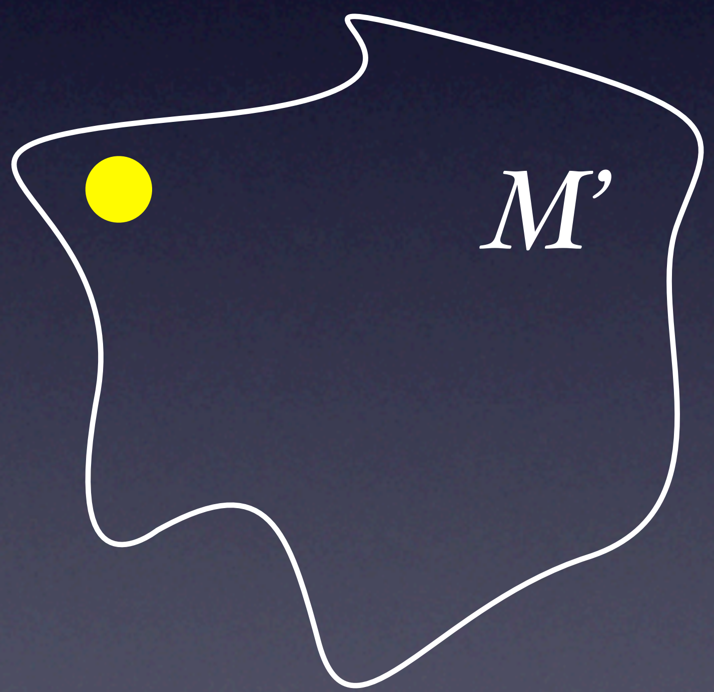

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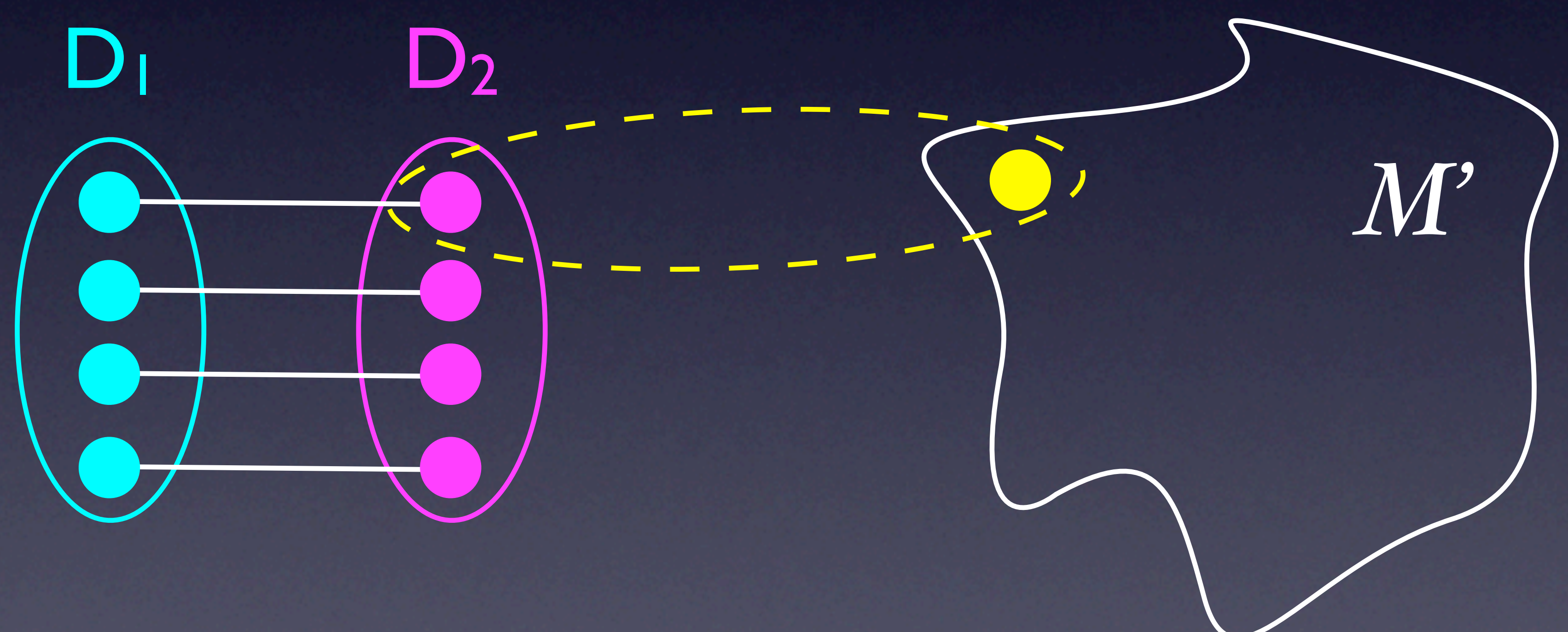

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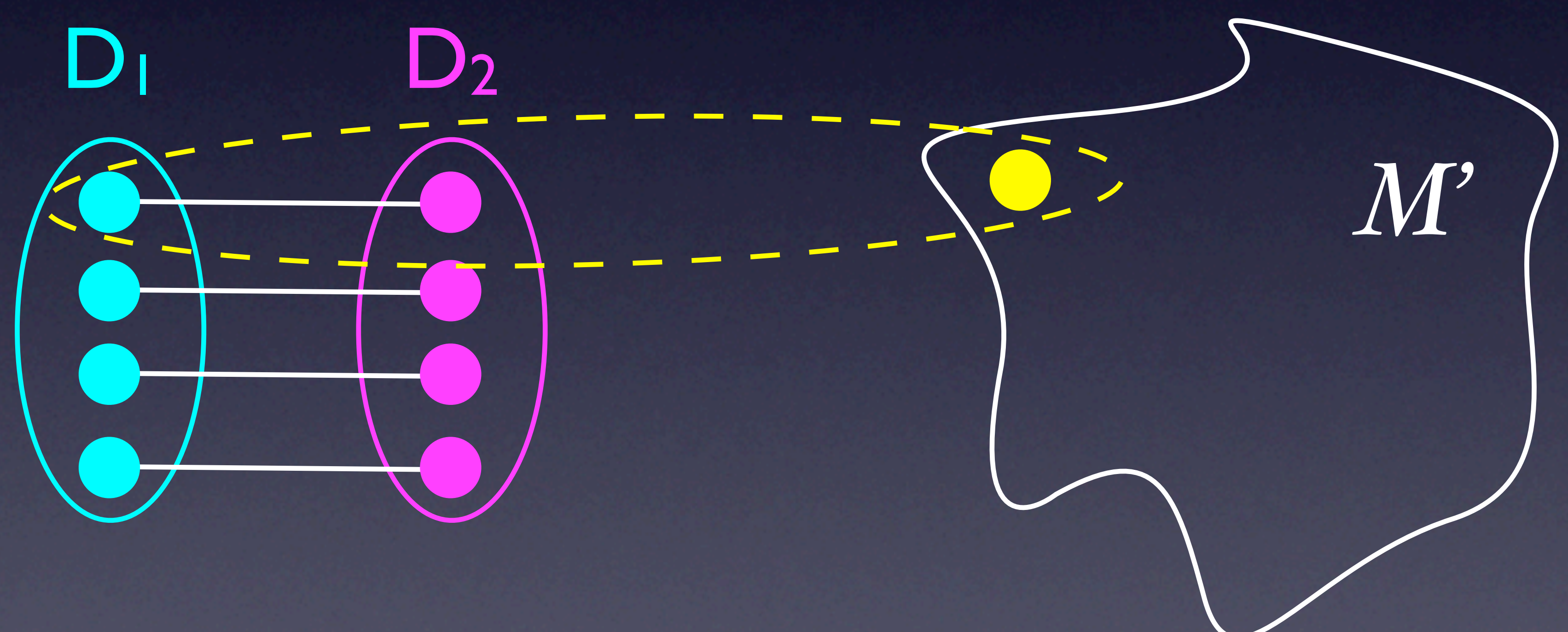

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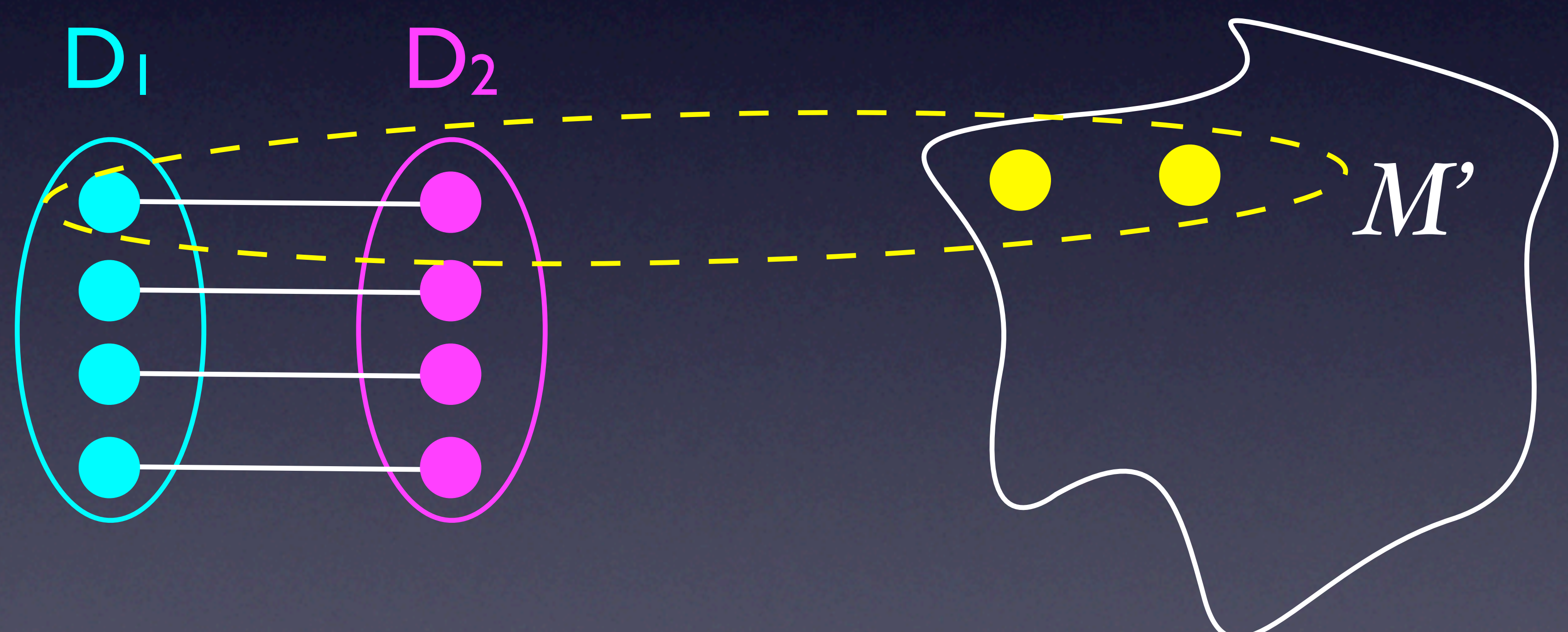

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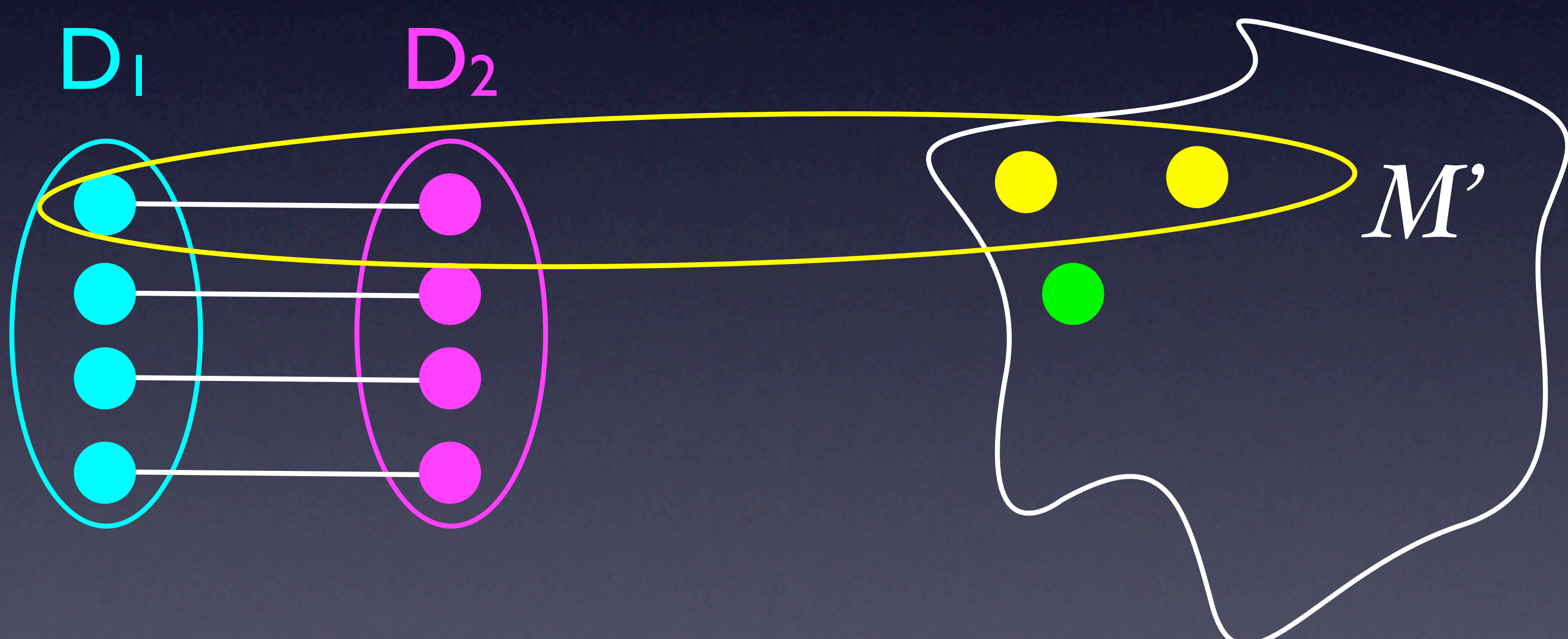

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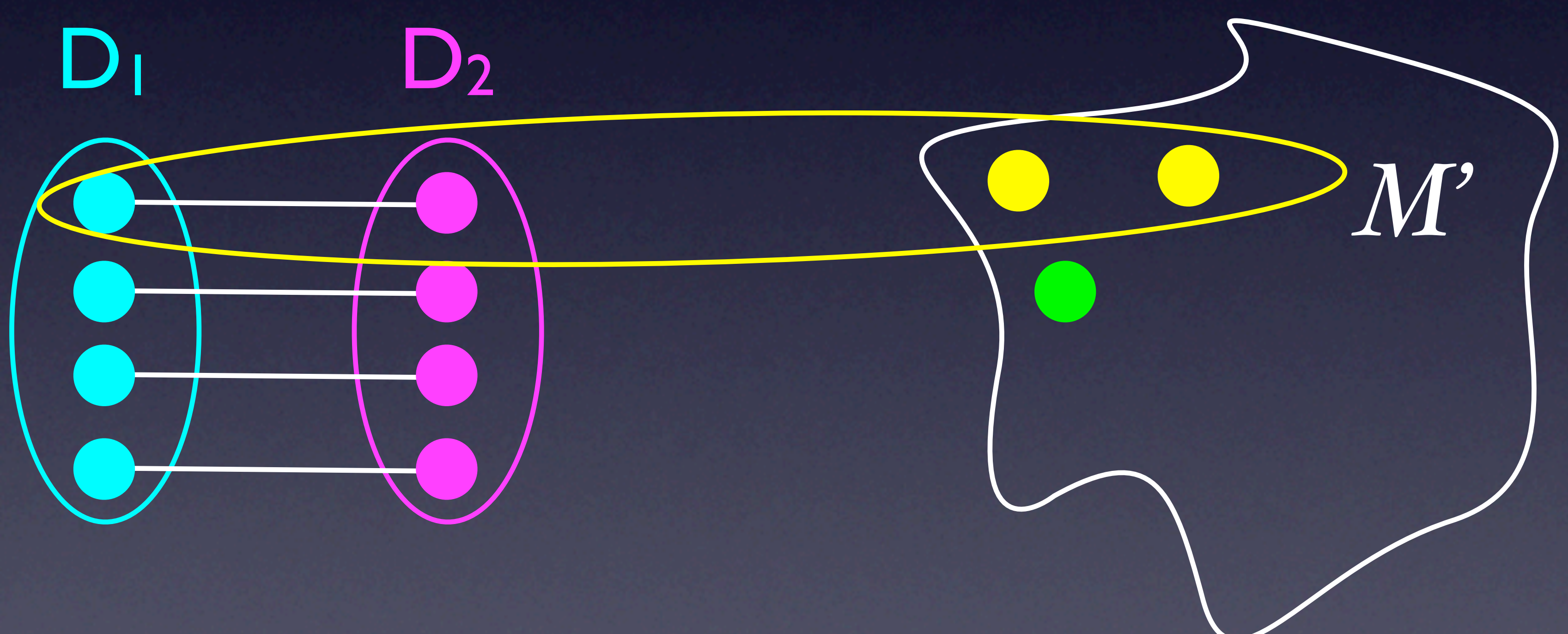

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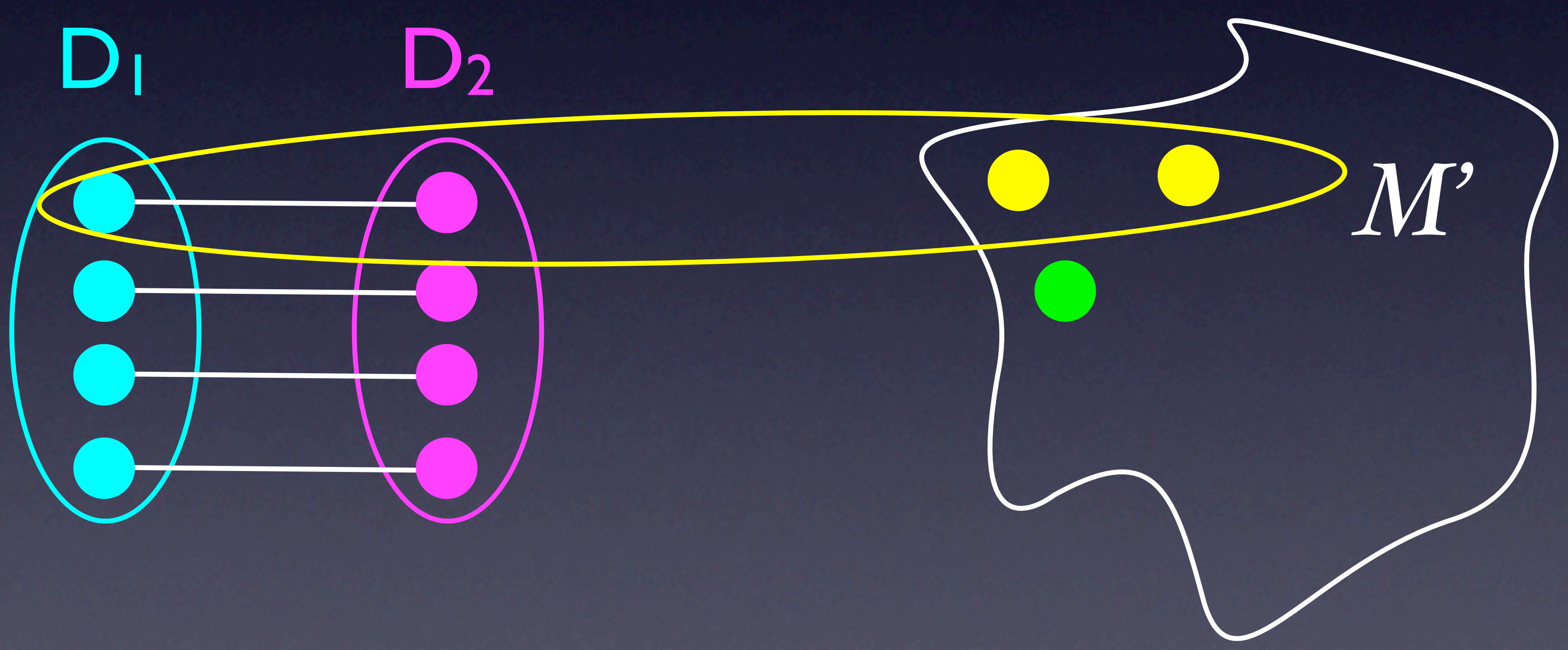

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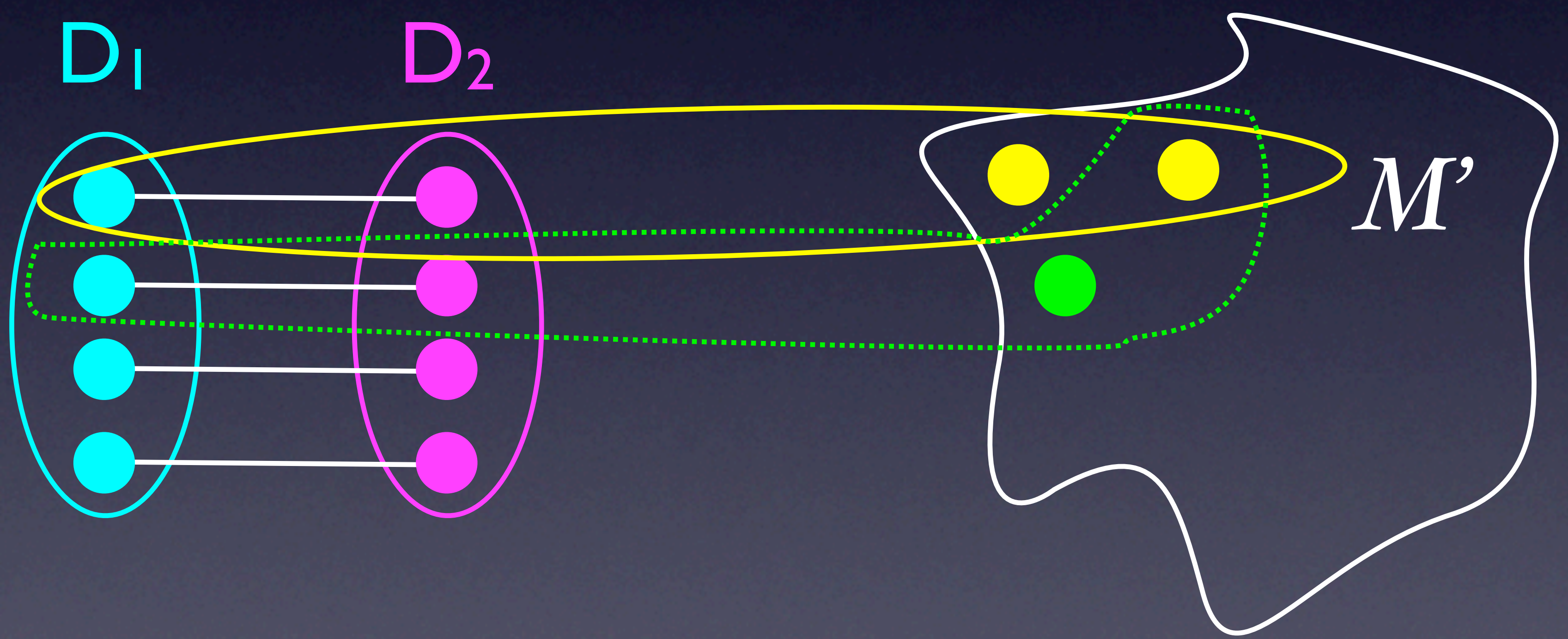

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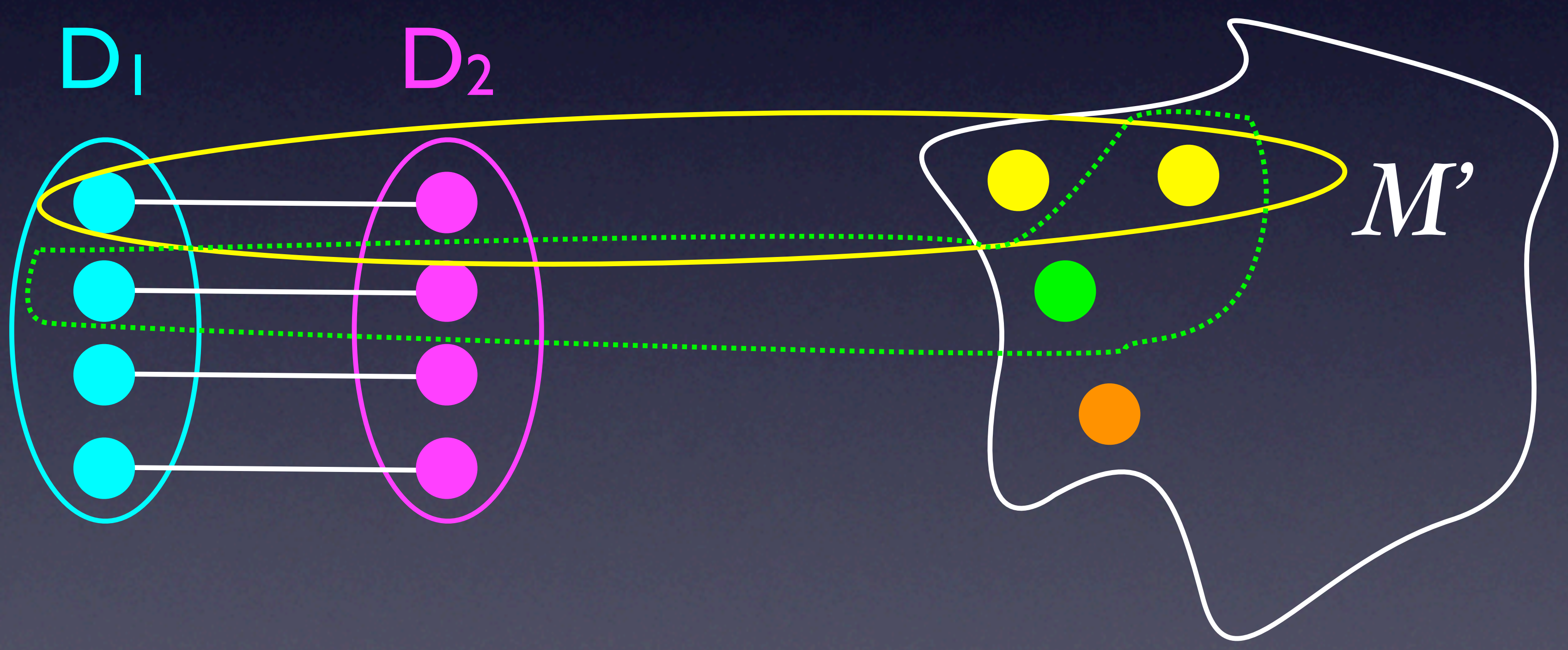

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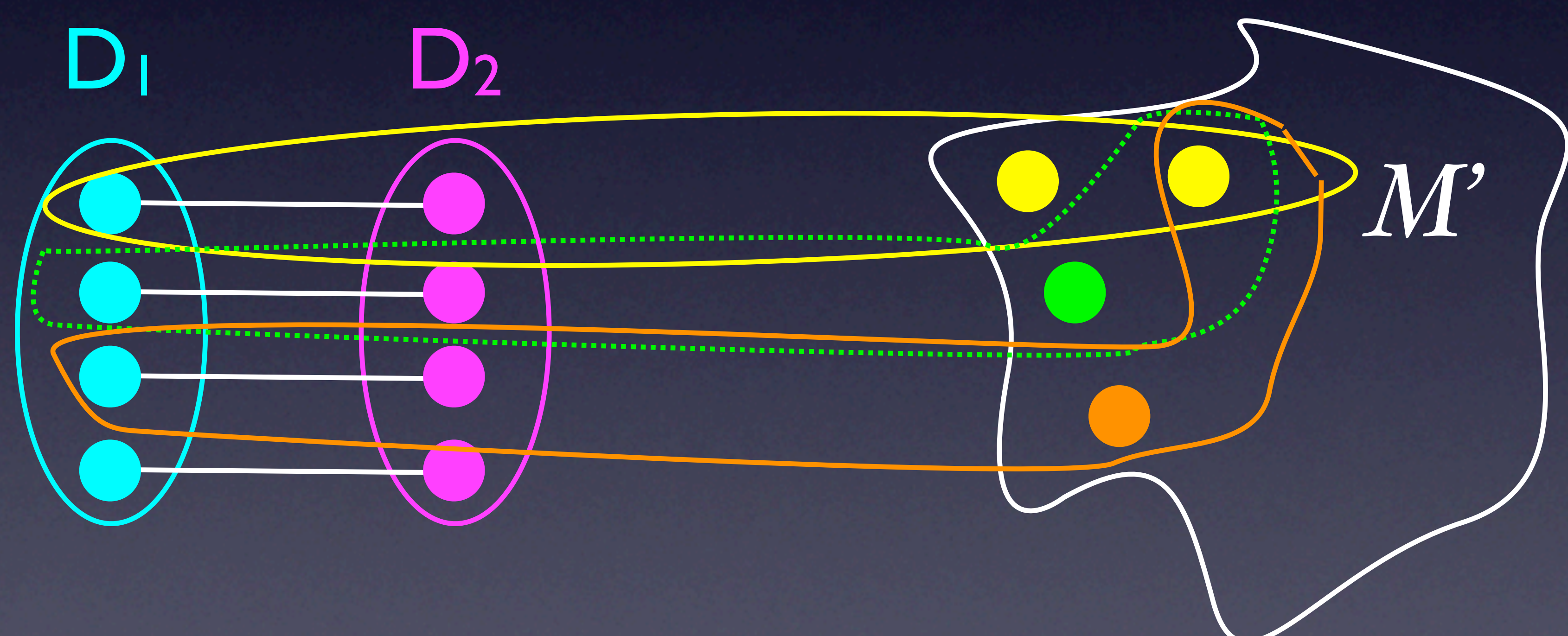

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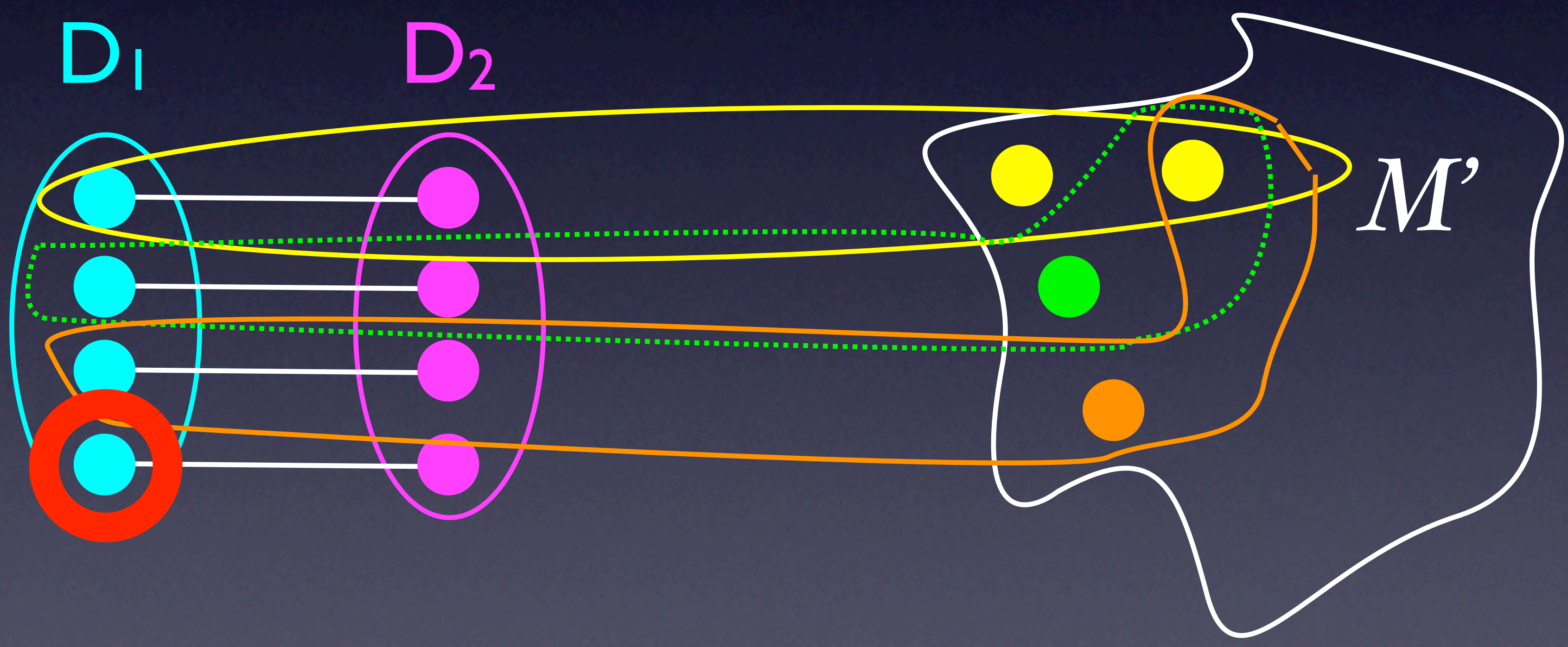

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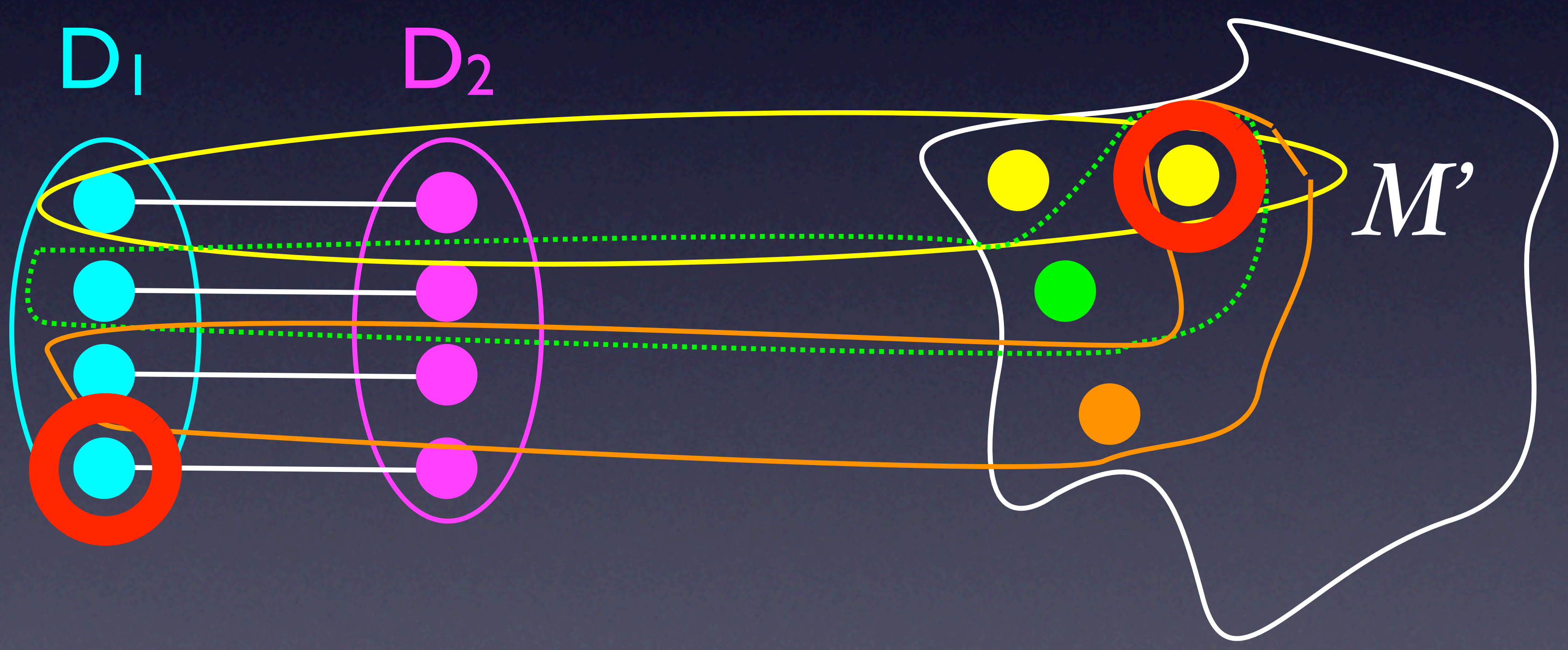

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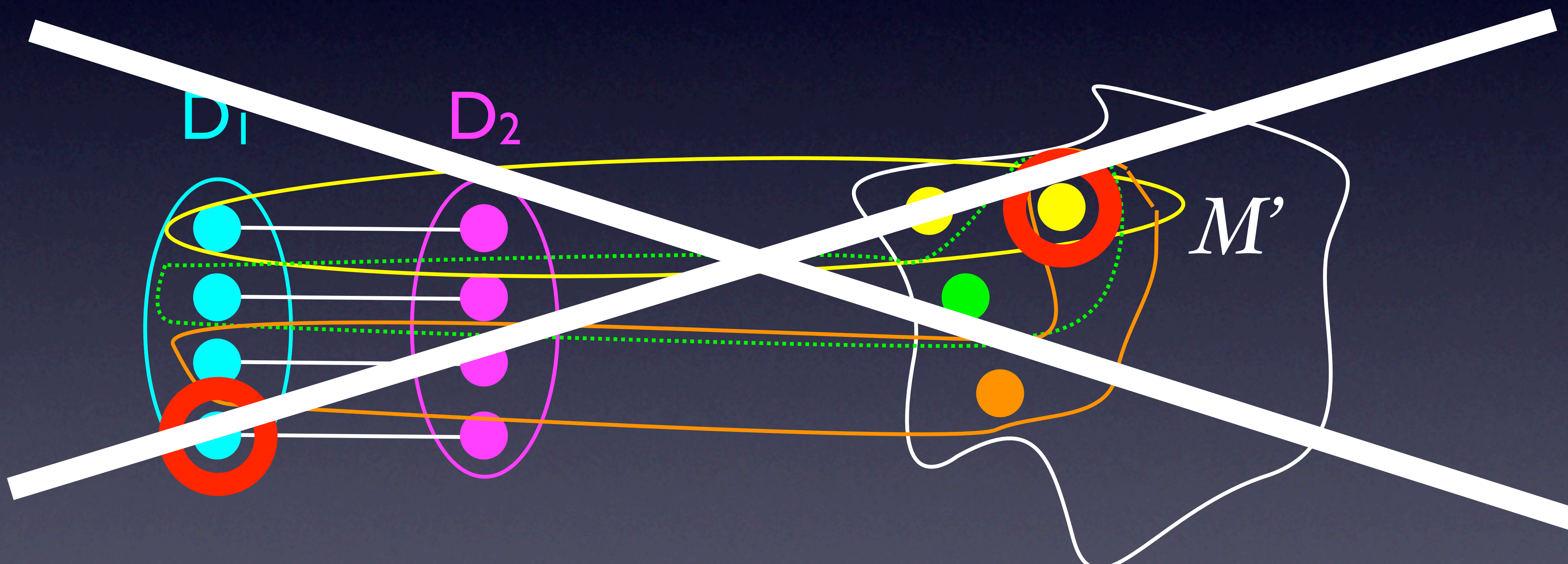

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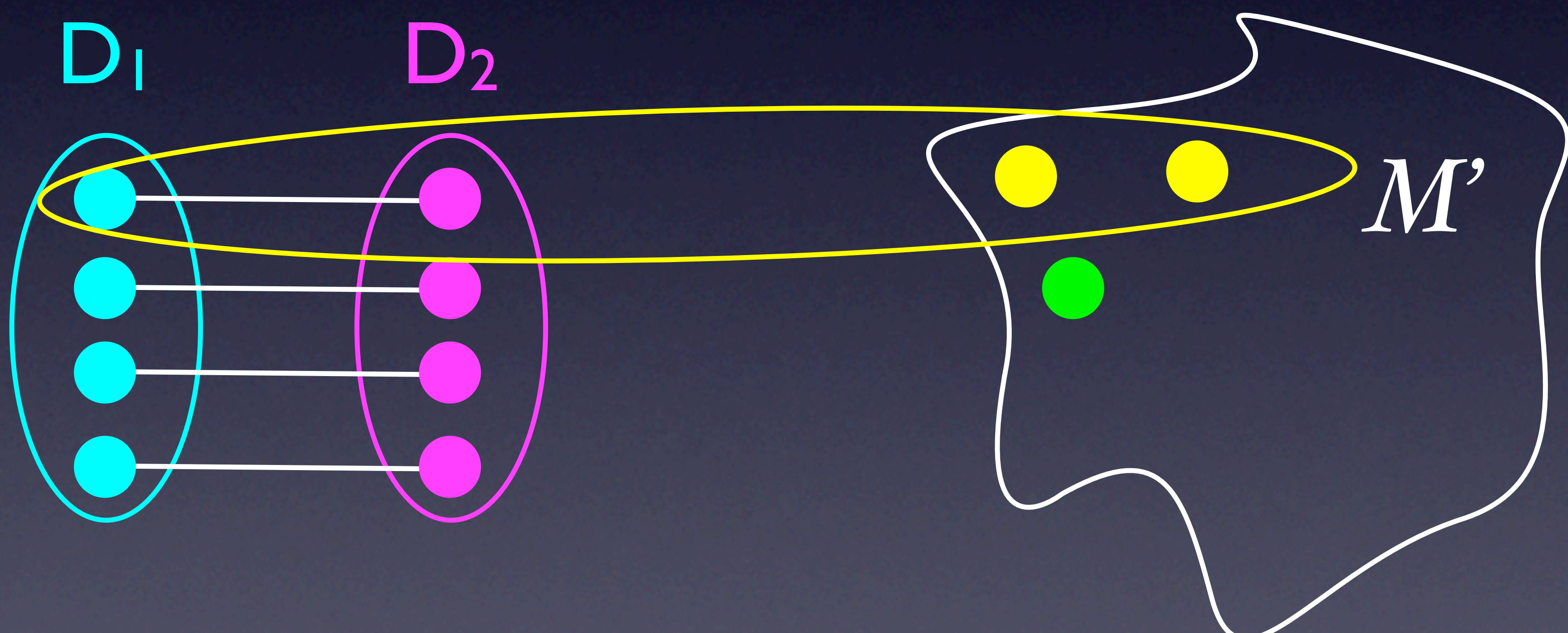

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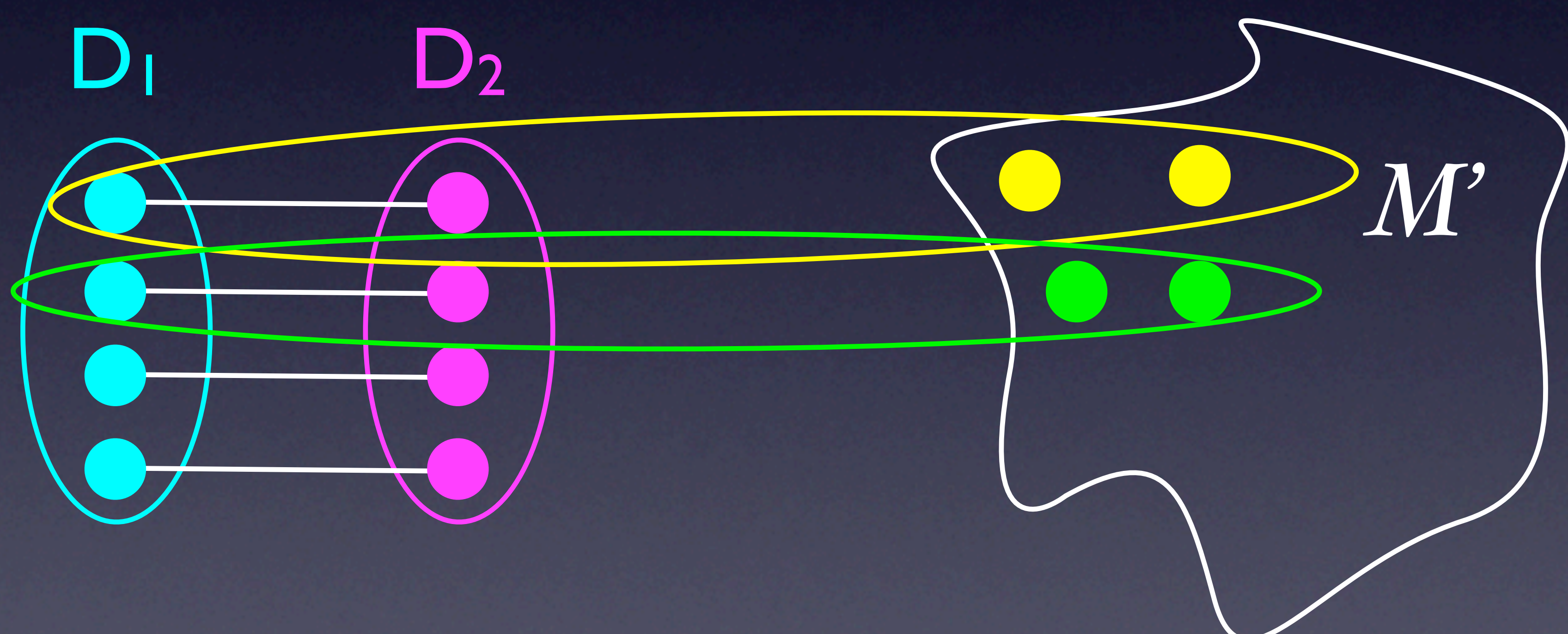

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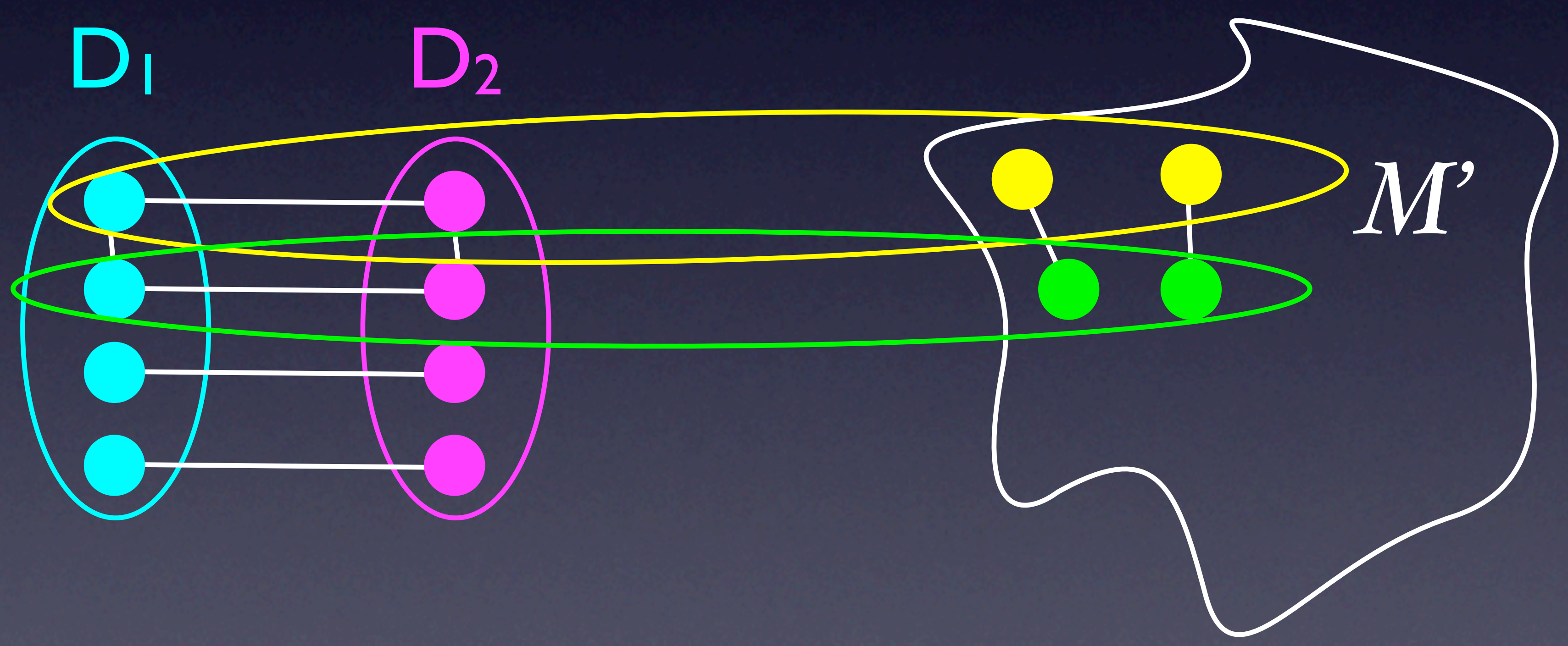

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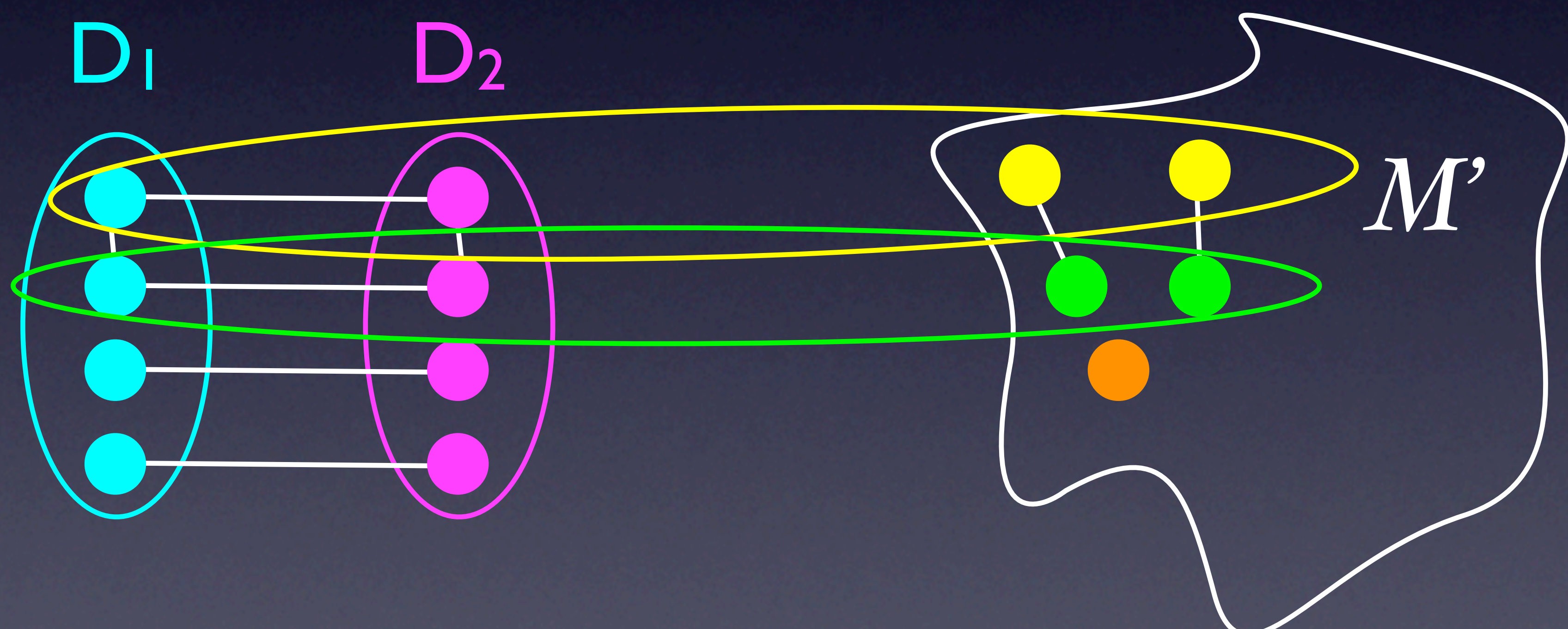

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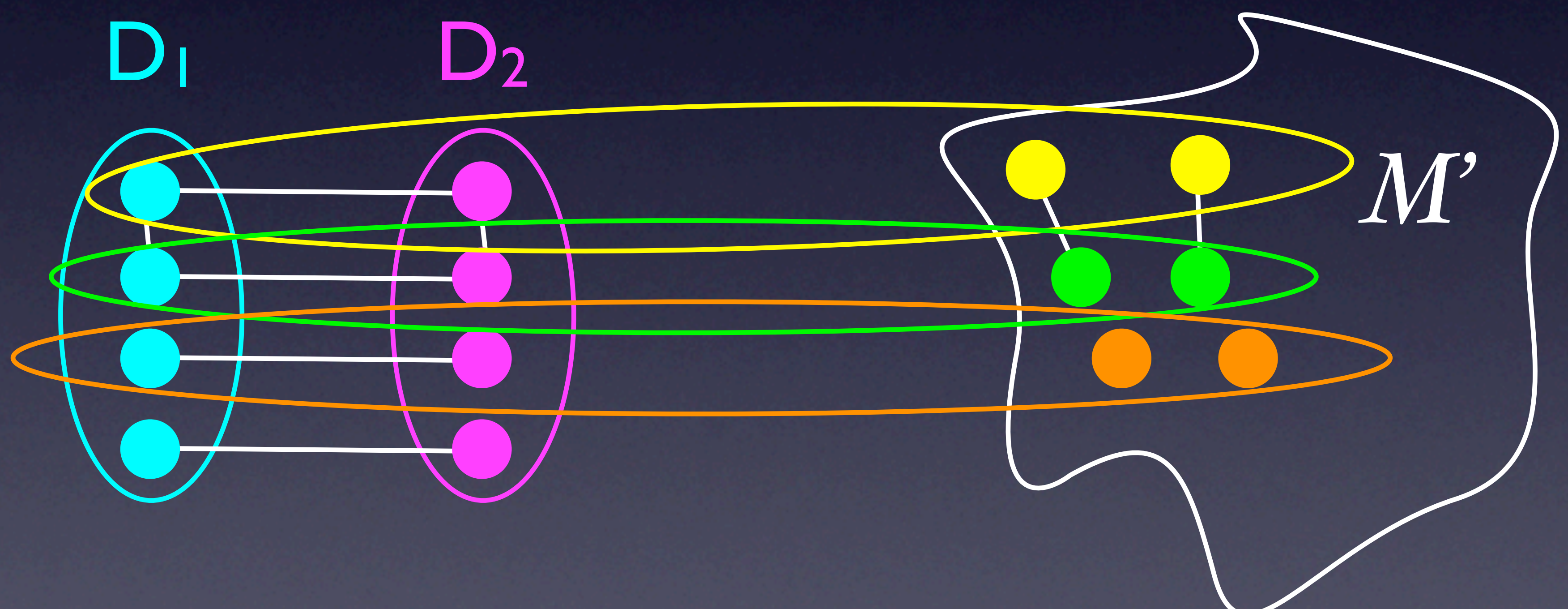

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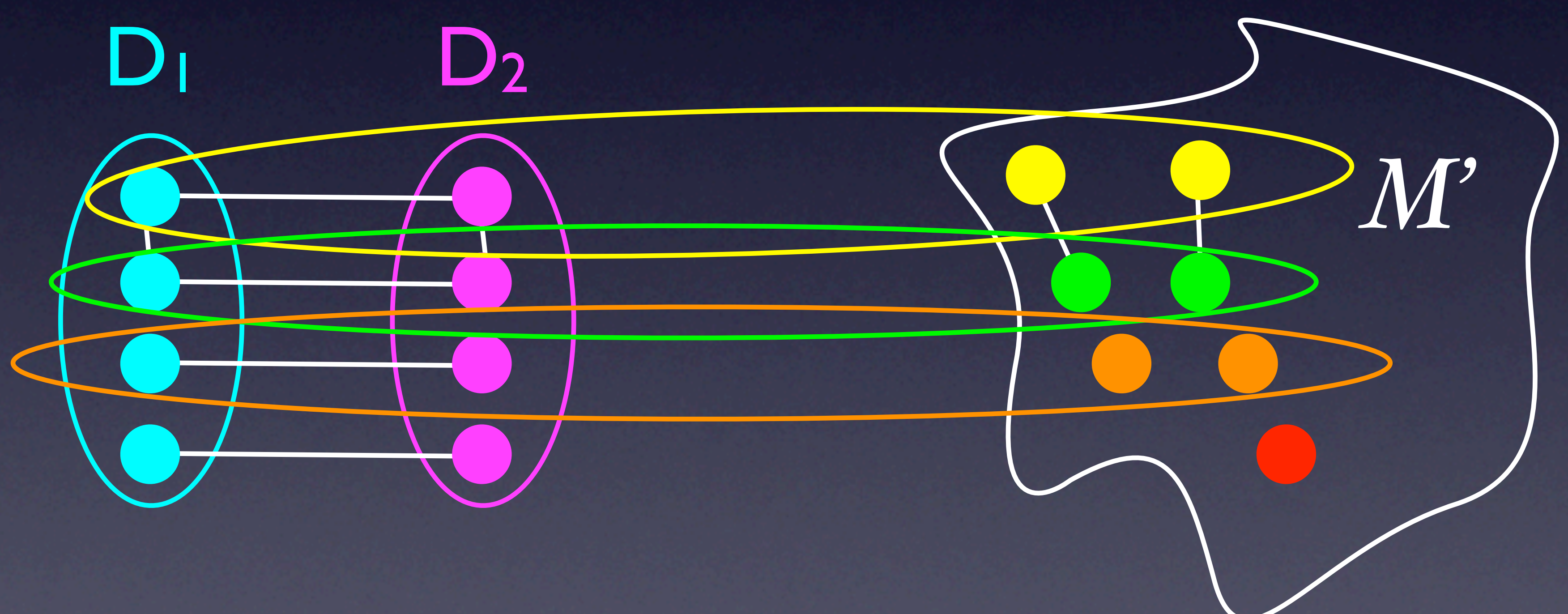

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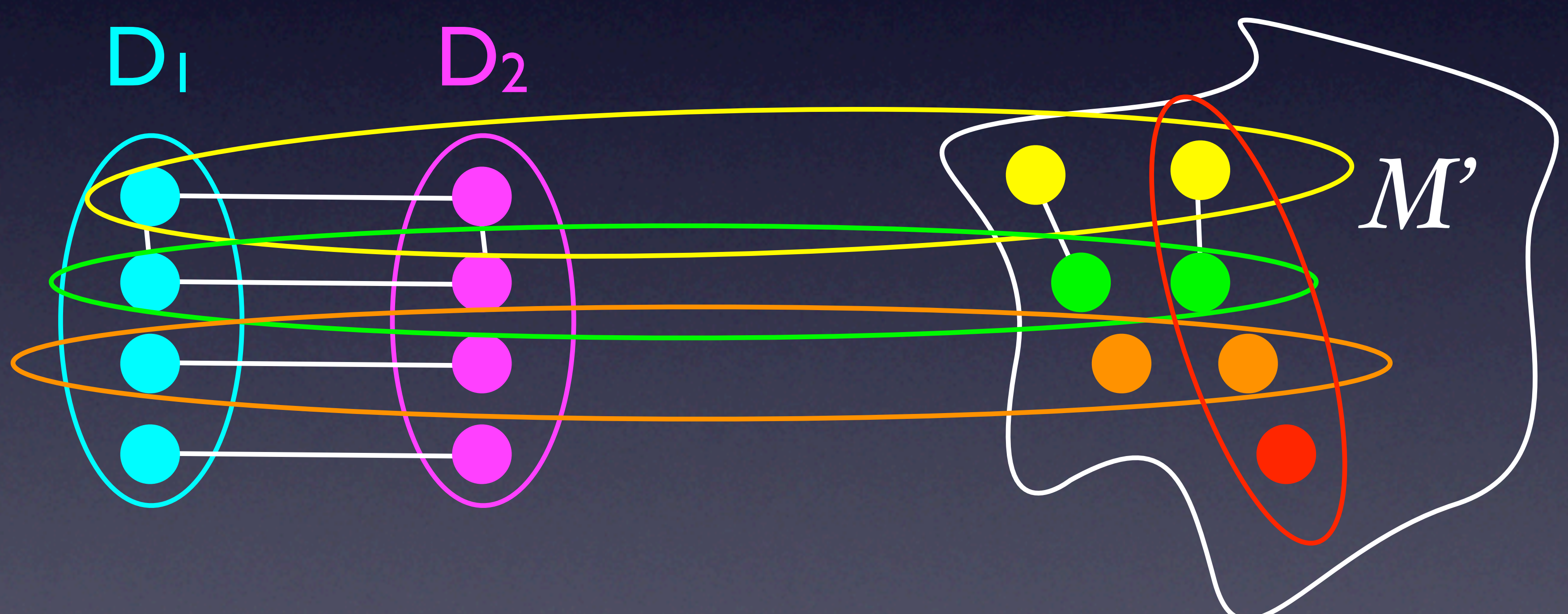

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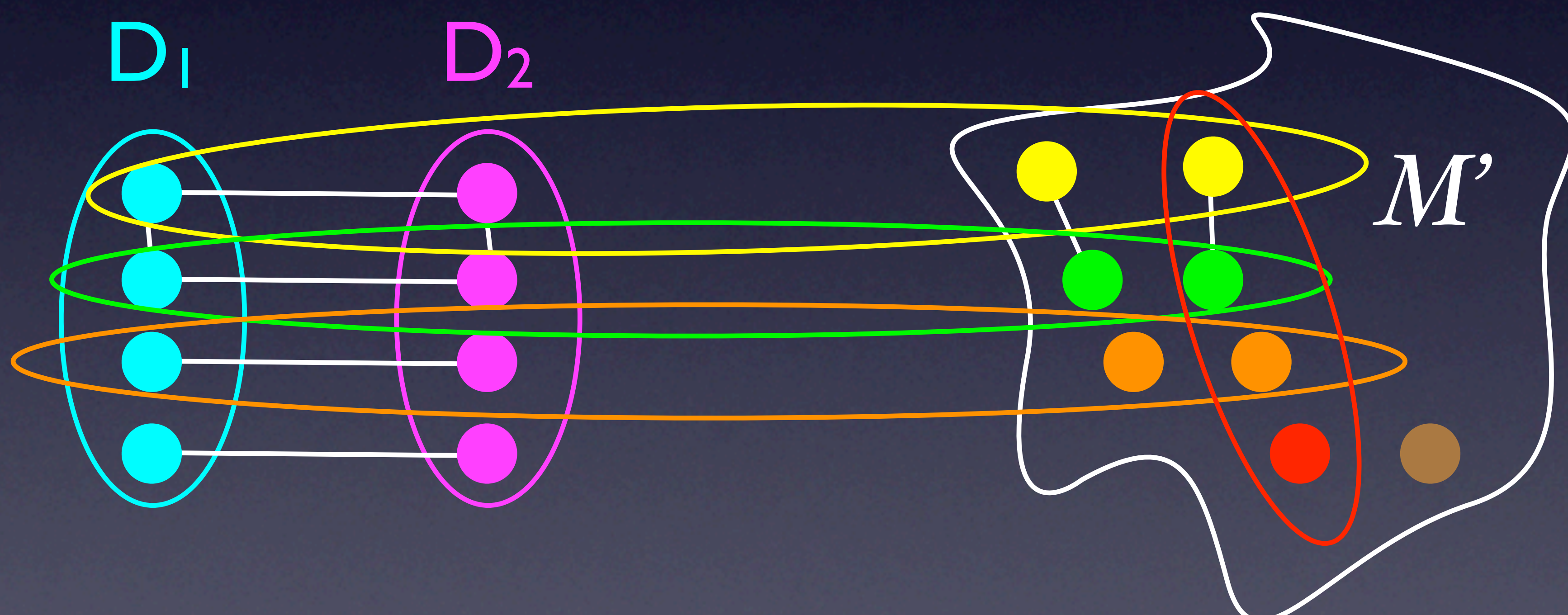

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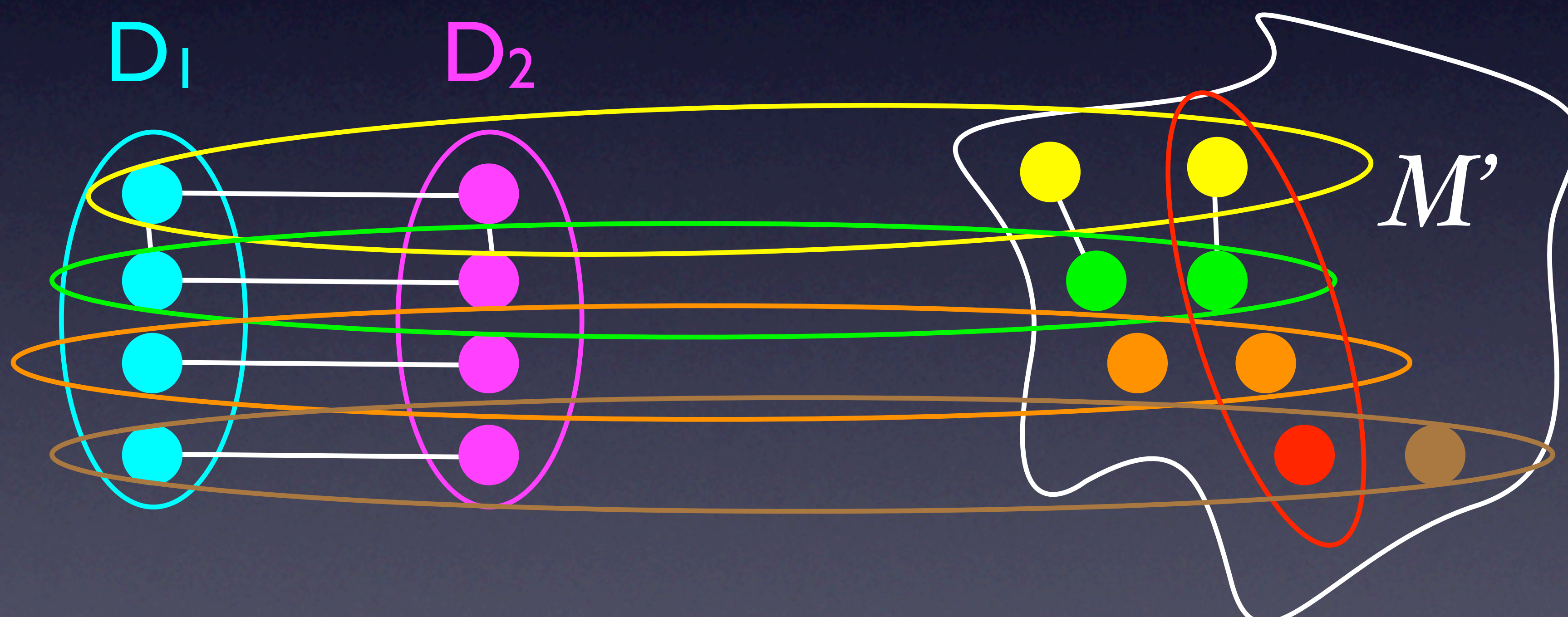

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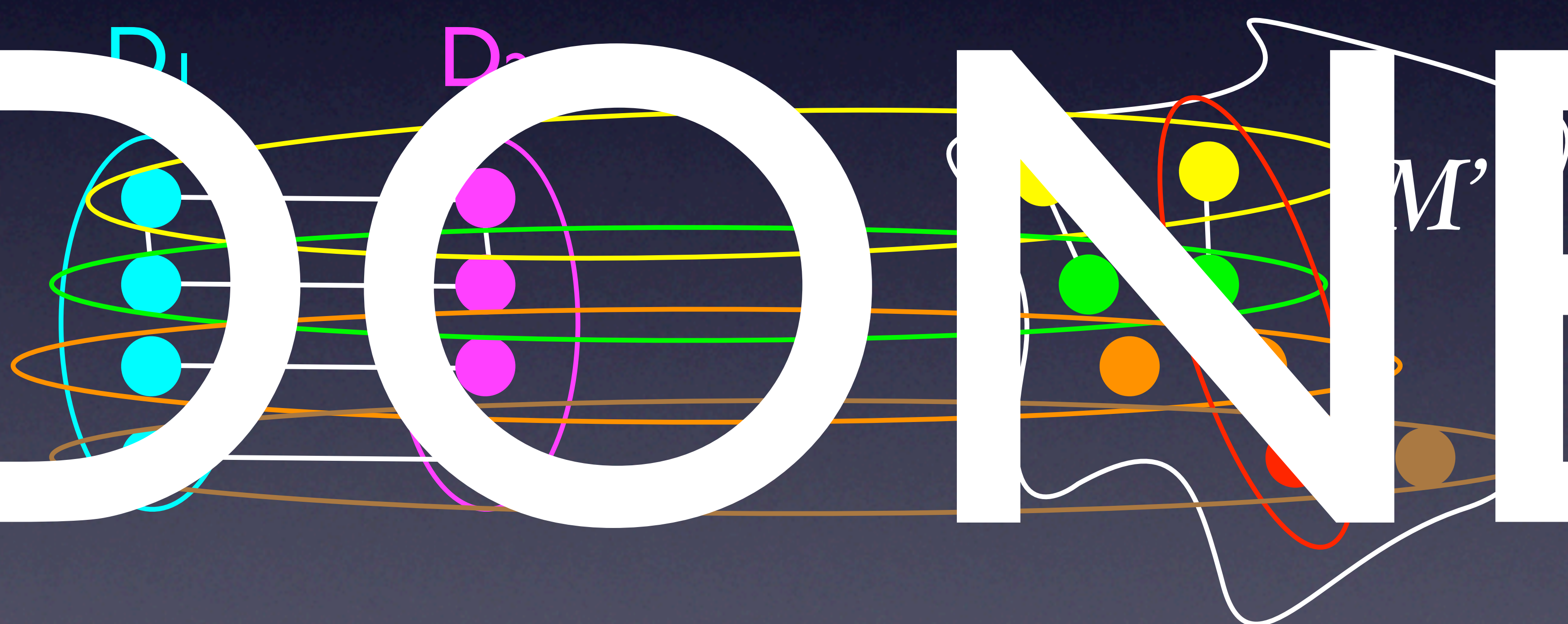

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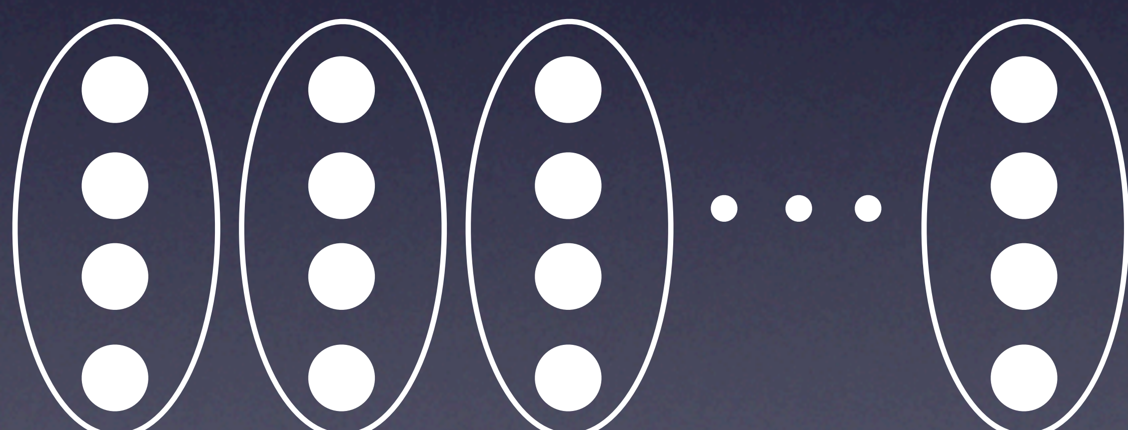
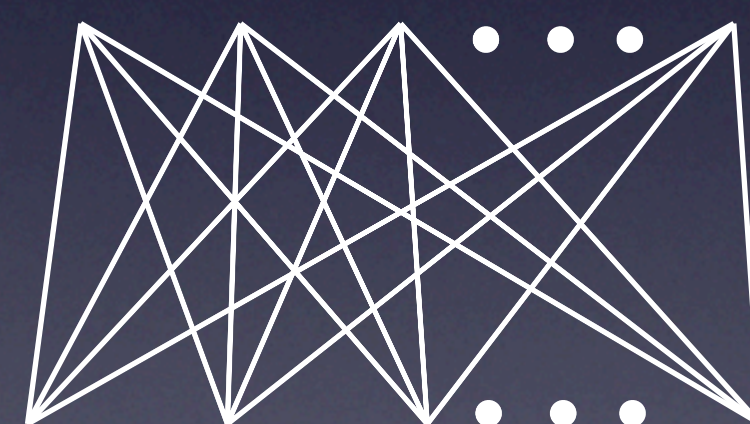


M



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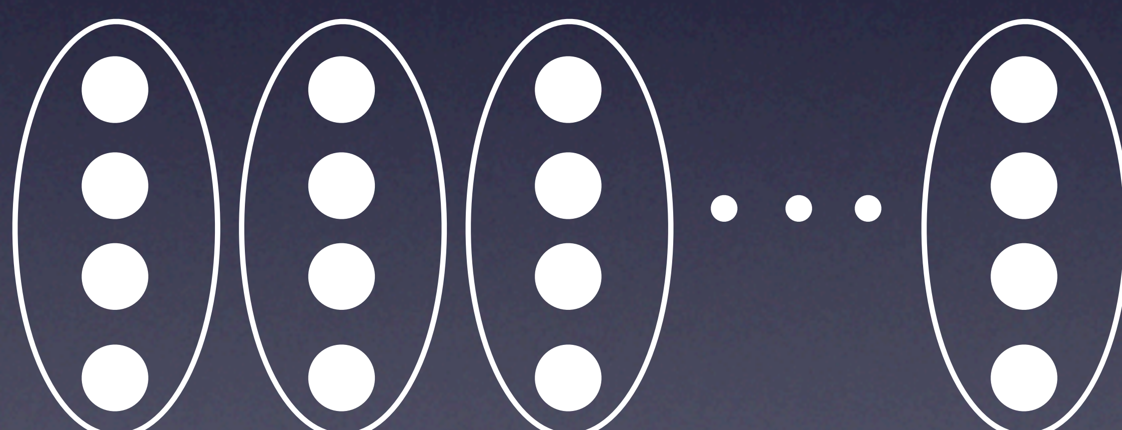
There is a minimal set, Z, that is a circuit in one but not the other.



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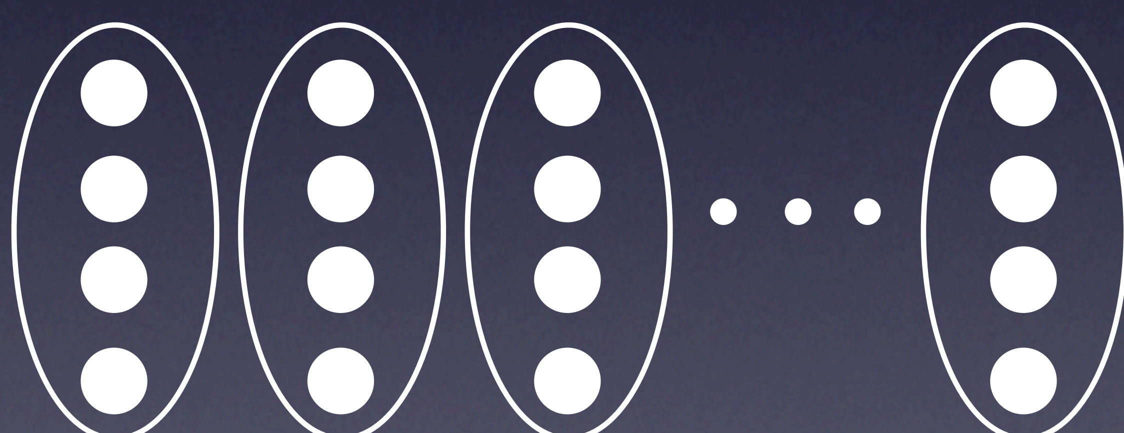

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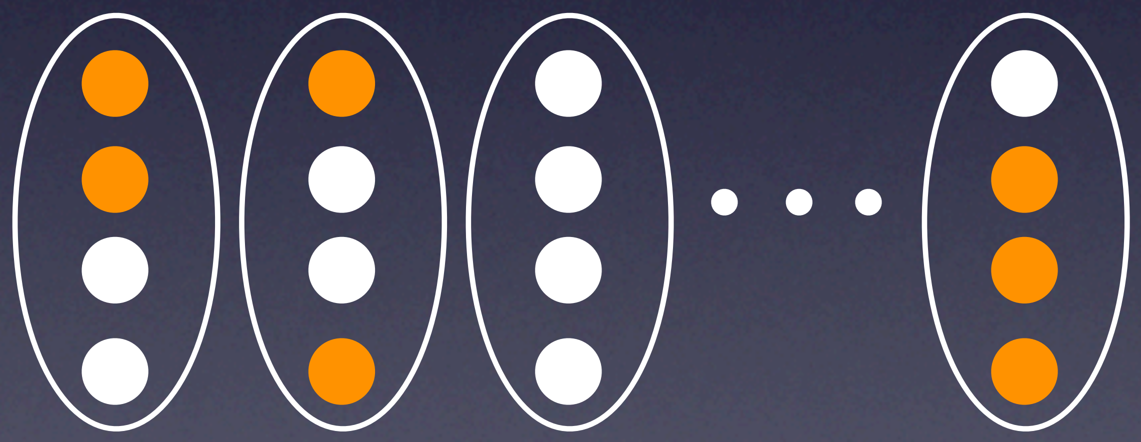

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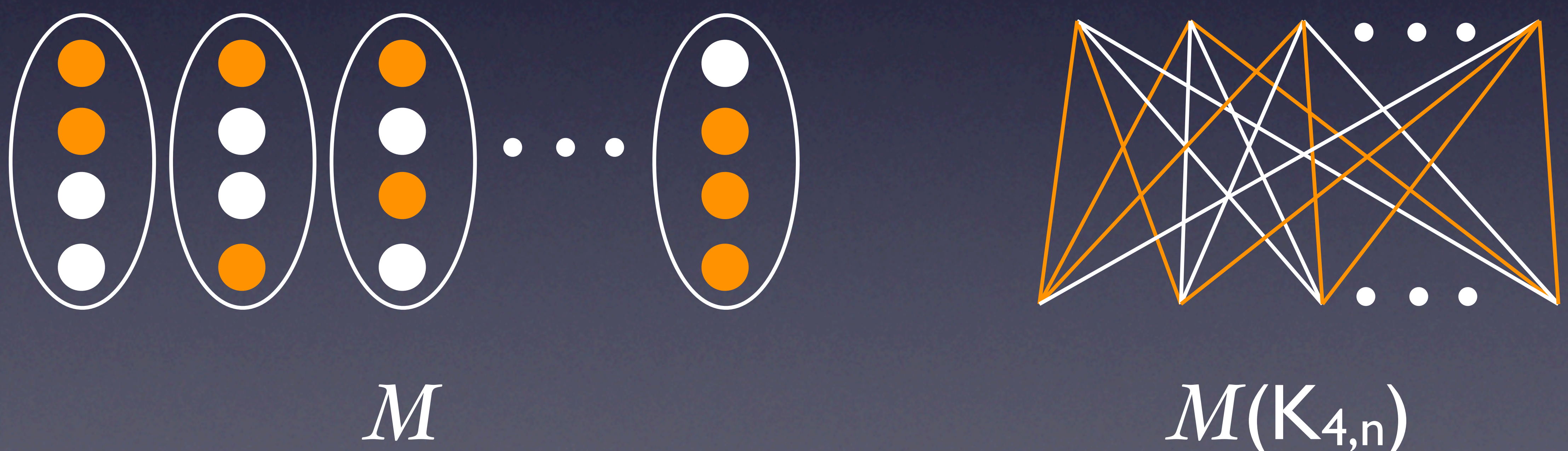

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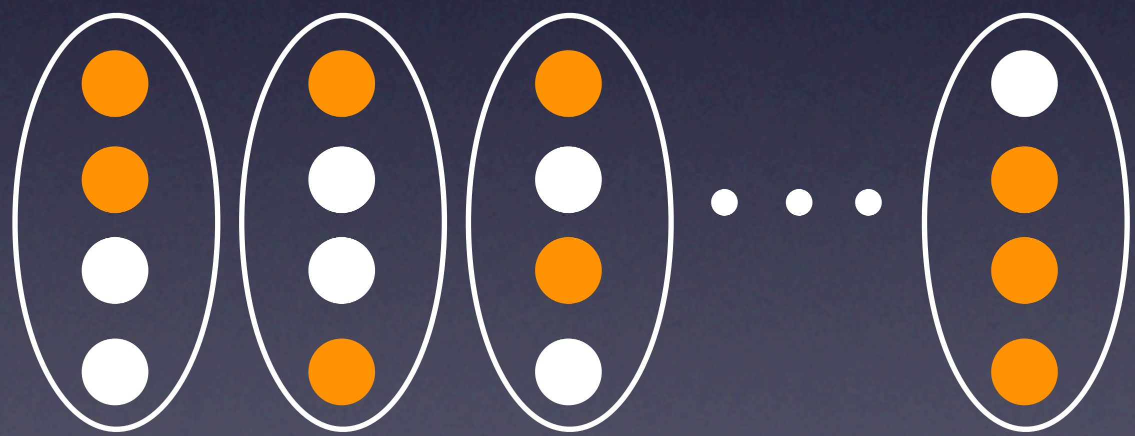

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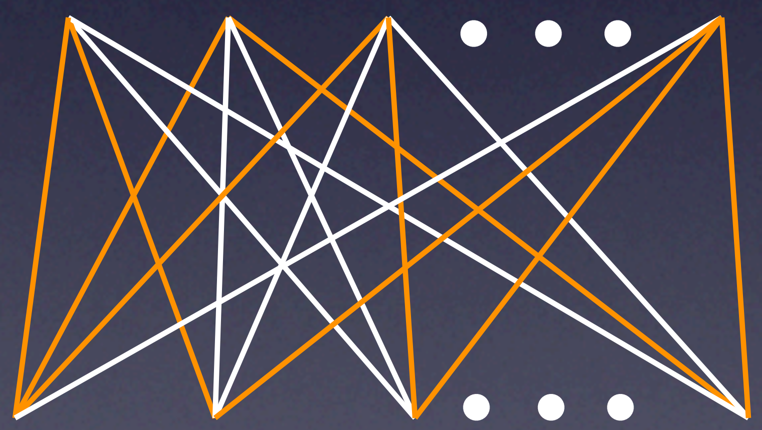

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The only 4-connected matroids in which every element is in both a 4-element circuit and a 4-element cocircuit are____________?