Unavoidable Induced Subgraphs of Large 2-Connected Graphs

Sarah Allred*
Guoli Ding
Bogdan Oporowski

Louisiana State University

7th Annual Mississippi Discrete Math Workshop

Background

Theorem (Ramsey (1928))

There is a function f such that every graph on $f(r)$ vertices contains an induced K_r or $\overline{K_r}$.

Background

Theorem (Ramsey (1928))

There is a function f such that every graph on $f(r)$ vertices contains an induced K_r or $\overline{K_r}$.

Theorem (Ramsey (1928))

Every giant graph contains a large induced K_r or $\overline{K_r}$

Background

Theorem (Ramsey (1928))

Every giant graph contains a large induced K_r or $\overline{K_r}$.

Theorem

Every giant **connected** *graph contains a large induced K_r, $K_{1,r}$, or P_r.*

Background

Theorem (Ramsey (1928))

Every giant graph contains a large induced K_r or $\overline{K_r}$.

Theorem

Every giant **connected** *graph contains a large induced K_r, $K_{1,r}$, or P_r.*

Theorem

*Every giant 2-***connected** *graph contains a* **subdivision** *of $K_{2,r}$ or C_r.*

What about 2-connected induced subgraphs?

Main Result

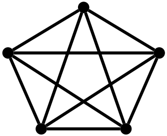
Theorem (A., D.,O. 2019+)

Every gargantuan 2-connected graph contains a large K_r, $TK_{2,r}$, $TK^+_{2,r}$, or CL_r as an induced subgraph.

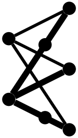
Main Result

Theorem (A., D.,O. 2019+)

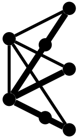
Every gargantuan 2-connected graph contains a large K_r, $TK_{2,r}$, $TK^+_{2,r}$, or CL_r as an induced subgraph.



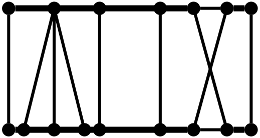
K_r



$TK_{2,r}$



$TK^+_{2,r}$

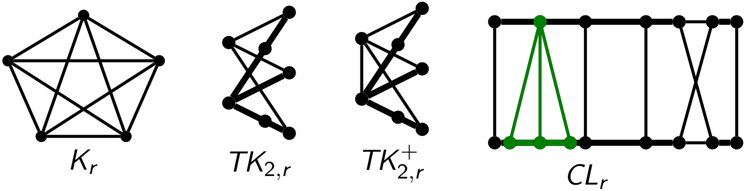


CL_r

Main Result

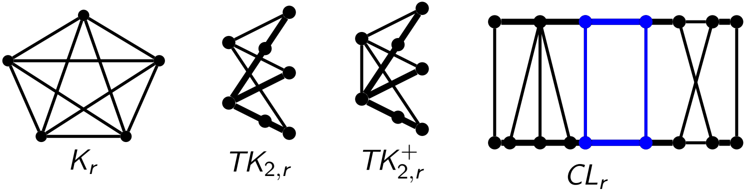
Theorem (A., D.,O. 2019+)

Every gargantuan 2-connected graph contains a large K_r, $TK_{2,r}$, $TK^{+}_{2,r}$, or CL_r as an induced subgraph.



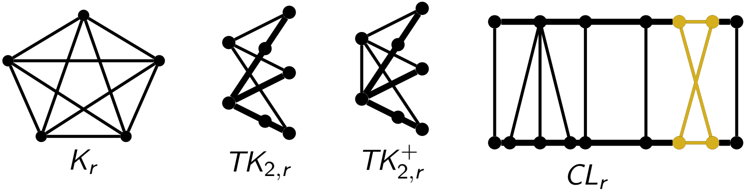
Theorem (A., D.,O. 2019+)

Every gargantuan 2-connected graph contains a large K_r, $TK_{2,r}$, $TK^+_{2,r}$, or CL_r as an induced subgraph.



Theorem (A., D.,O. 2019+)

Every gargantuan 2-connected graph contains a large K_r, $TK_{2,r}$, $TK^+_{2,r}$, or CL_r as an induced subgraph.



Graphs without a long path

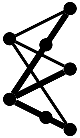
Lemma (A.,D.,O.,2019+)

Every giant 2-connected graph contains either a long path or an induced $TK_{2,r}$ *or* $TK^+_{2,r}$.

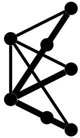
Graphs without a long path

Lemma (A.,D.,O.,2019+)

Every giant 2-connected graph contains either a long path or an induced $TK_{2,r}$ or $TK_{2,r}^{+}$.

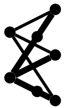


$TK_{2,r}$

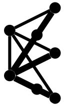


$TK_{2,r}^{+}$

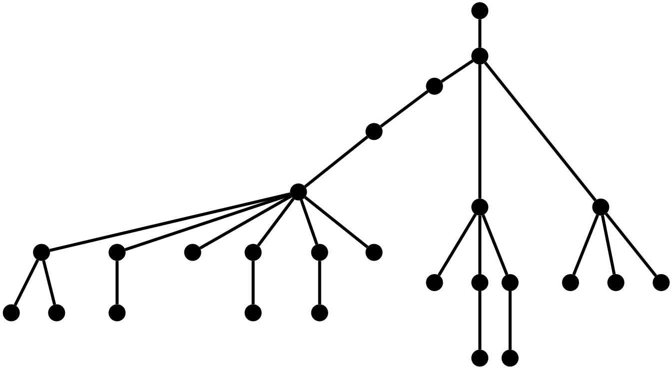
Graphs without a long path



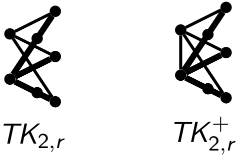
$TK_{2,r}$

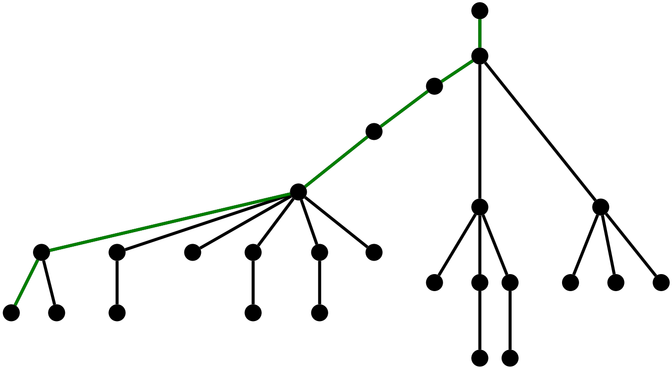


$TK^+_{2,r}$

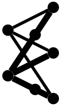


Graphs without a long path

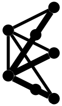




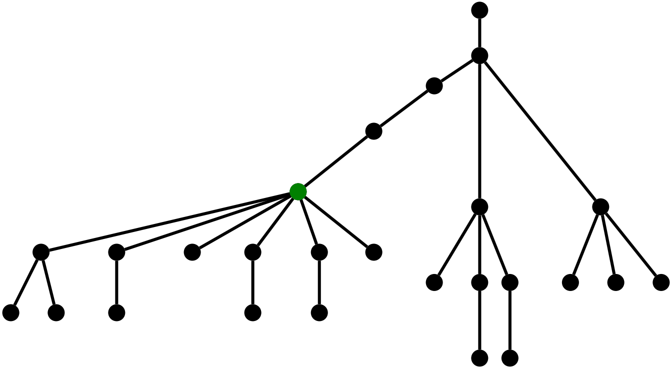
Graphs without a long path



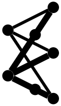
$TK_{2,r}$



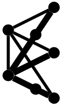
$TK_{2,r}^{+}$



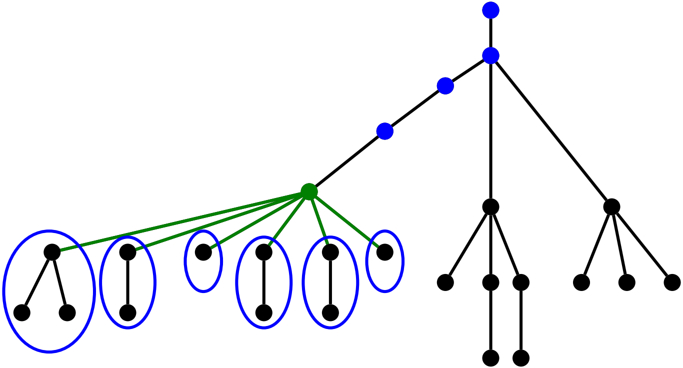
Graphs without a long path

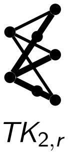


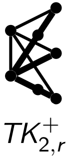
$TK_{2,r}$

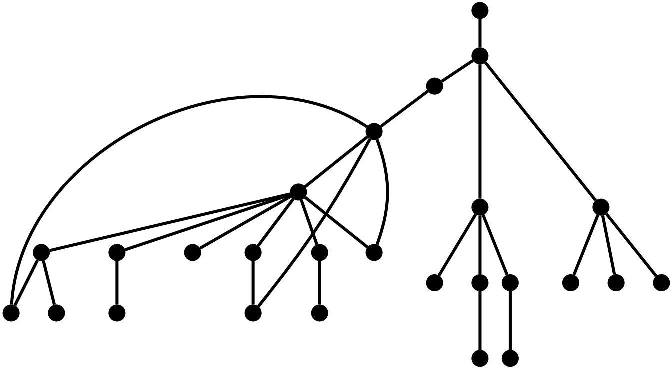


$TK^+_{2,r}$

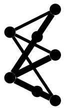



$TK_{2,r}$

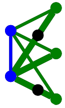

$TK^+_{2,r}$



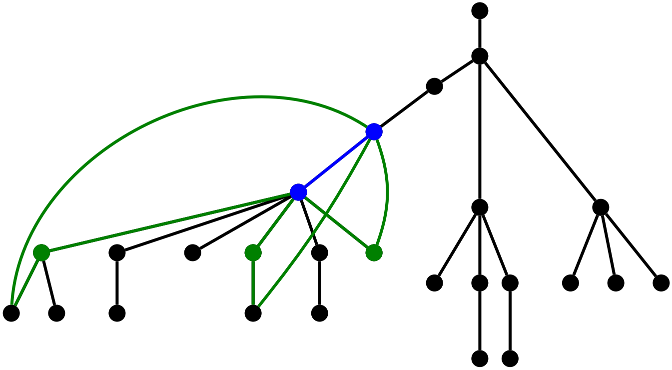
Graphs without a long path

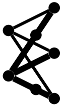


$TK_{2,r}$

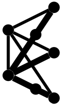


$TK_{2,r}^{+}$

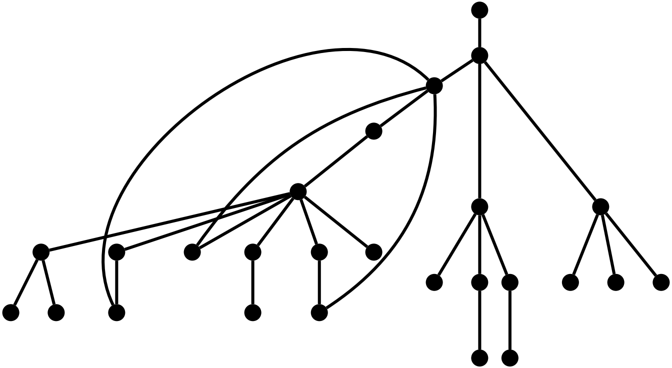




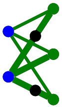
$TK_{2,r}$



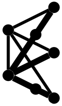
$TK^{+}_{2,r}$



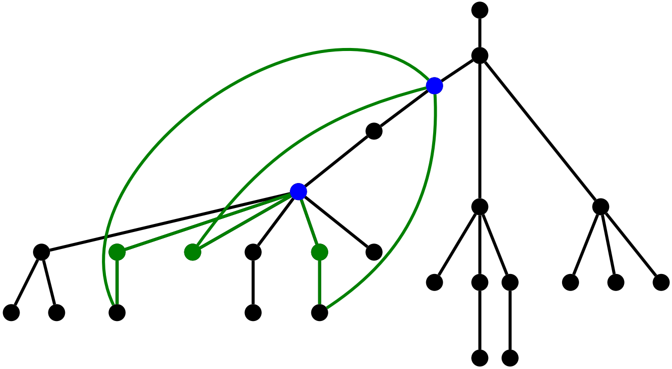
Graphs without a long path



$TK_{2,r}$



$TK^{+}_{2,r}$



Graphs with long path

Theorem (Galvin, Rival, Sands)

Every graph with a really long path contains a large induced K_r, $K_{r,r}$, or P_r.

Graphs with long path

Theorem (Galvin, Rival, Sands)

Every graph with a really long path contains a large induced K_r, $K_{r,r}$, or P_r.

Lemma (A., D.,O. 2019+)

Every 2-connected graph with a really long path contains a large induced K_r, $K_{r,r}$, or long messy ladder.

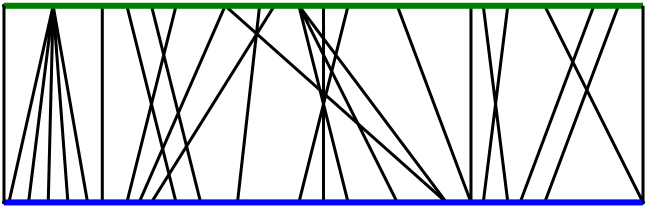
Graphs with long path

Theorem (Galvin, Rival, Sands)

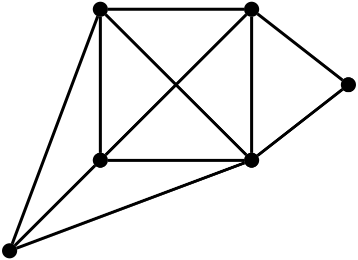
Every graph with a really long path contains a large induced K_r, $K_{r,r}$, or P_r.

Lemma (A., D.,O. 2019+)

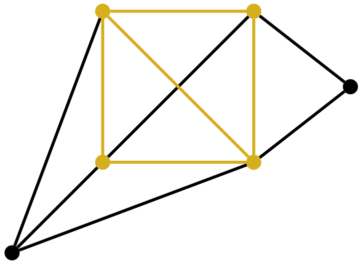
Every 2-connected graph with a really long path contains a large induced K_r, $K_{r,r}$, or long messy ladder.



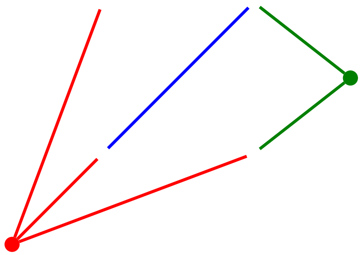
Bridges (Tutte)



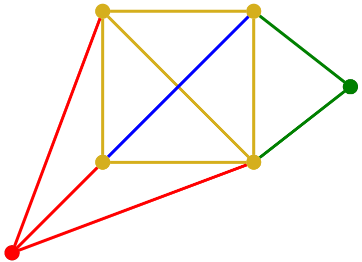
Bridges (Tutte)



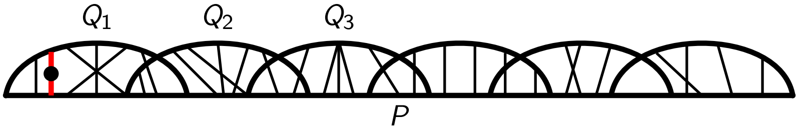
Bridges (Tutte)



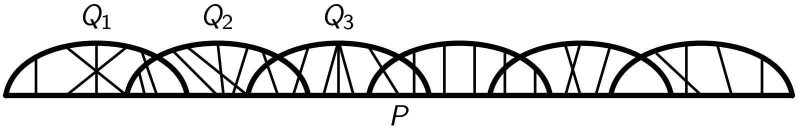
Bridges (Tutte)



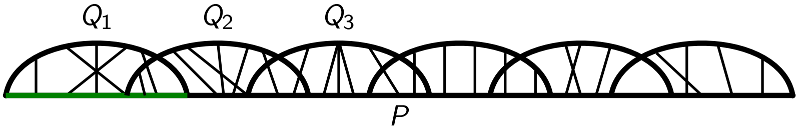
Long Path to Messy Ladder



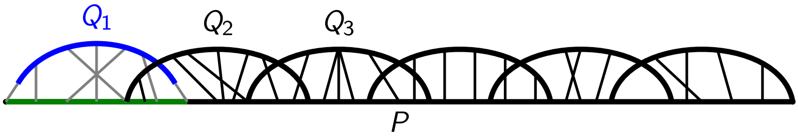
Long Path to Messy Ladder



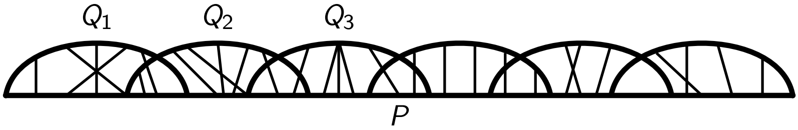
Long Path to Messy Ladder



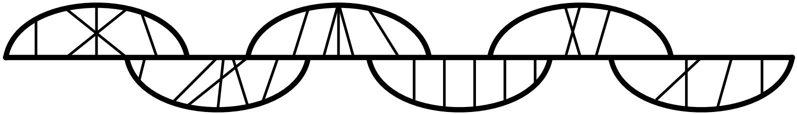
Long Path to Messy Ladder



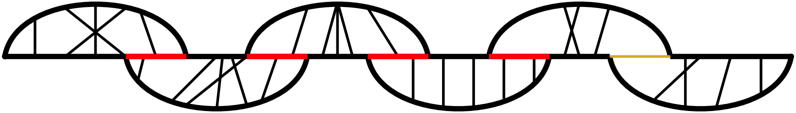
Long Path to Messy Ladder



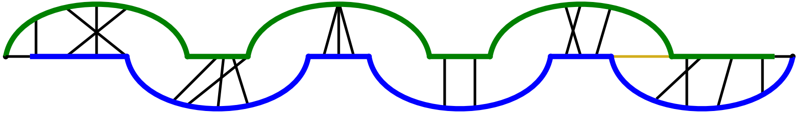
Long Path to Messy Ladder



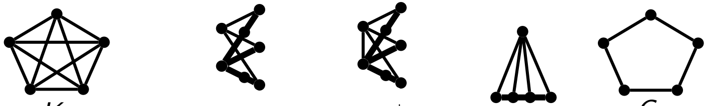
Long Path to Messy Ladder



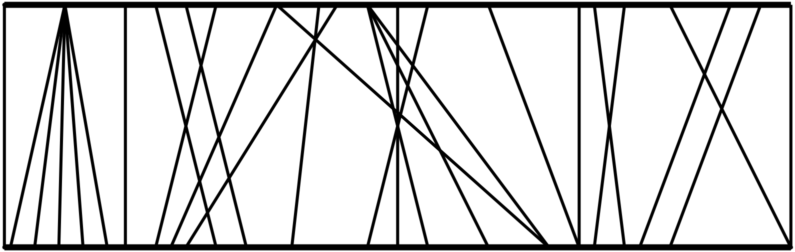
Long Path to Messy Ladder



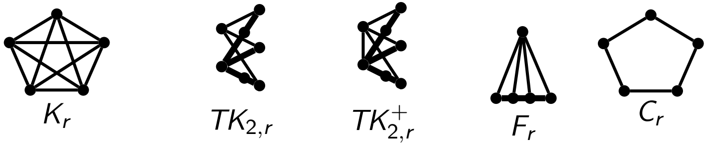
Messy Ladder to Clean Ladder

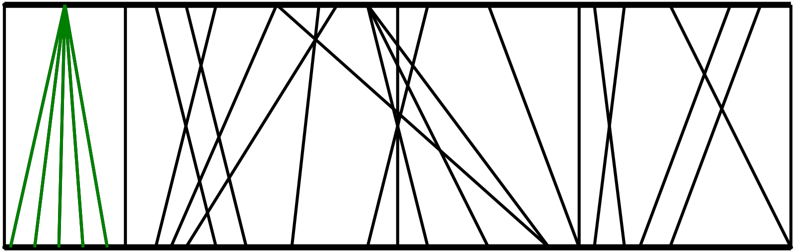


K_r $TK_{2,r}$ $TK^+_{2,r}$ F_r C_r

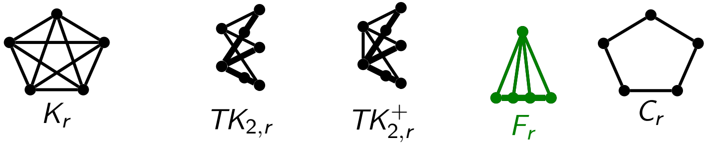


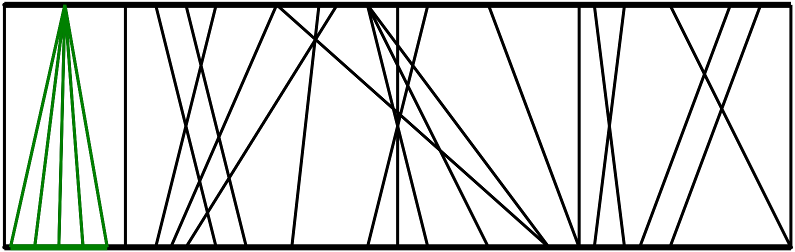
Messy Ladder to Clean Ladder



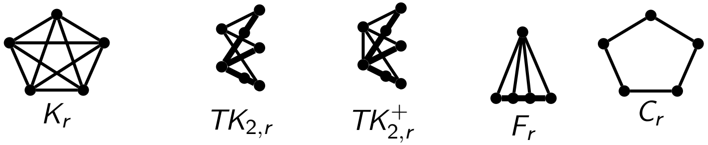


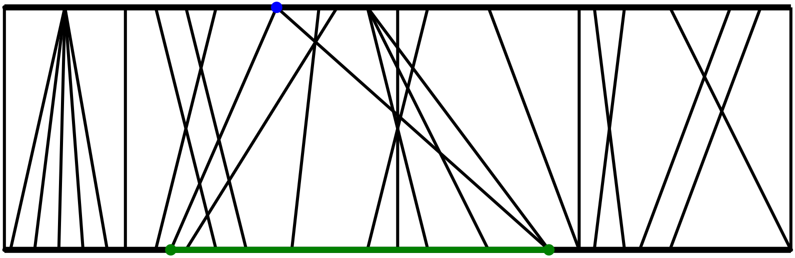
Messy Ladder to Clean Ladder



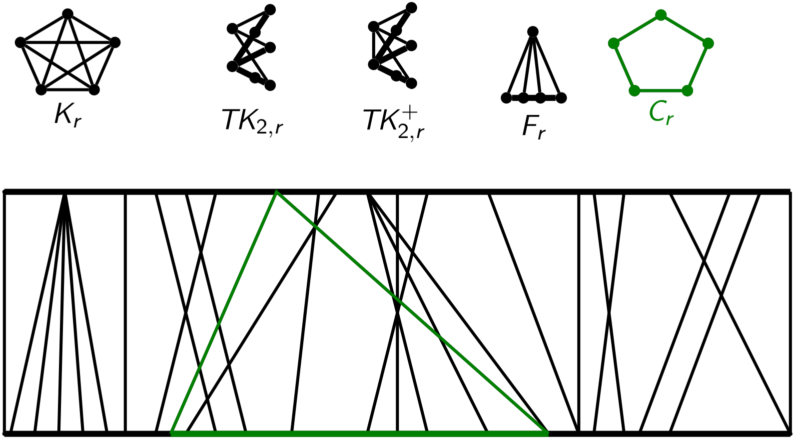


Messy Ladder to Clean Ladder

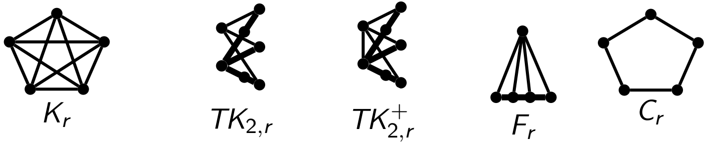


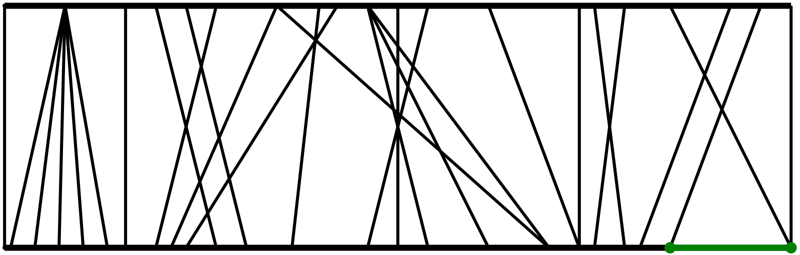


Messy Ladder to Clean Ladder

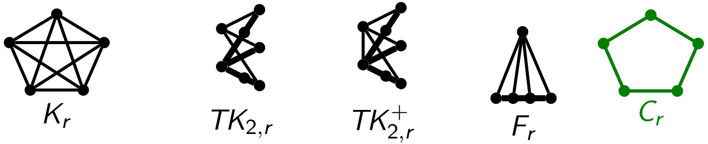


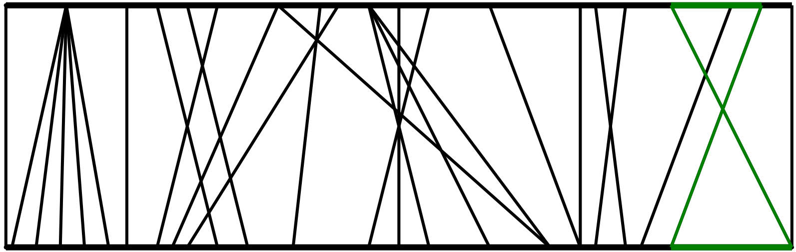
Messy Ladder to Clean Ladder





Messy Ladder to Clean Ladder

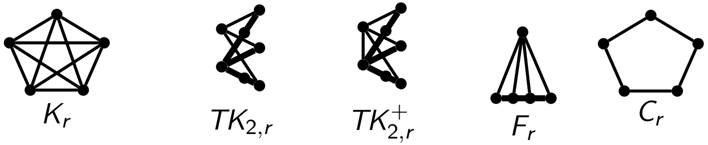


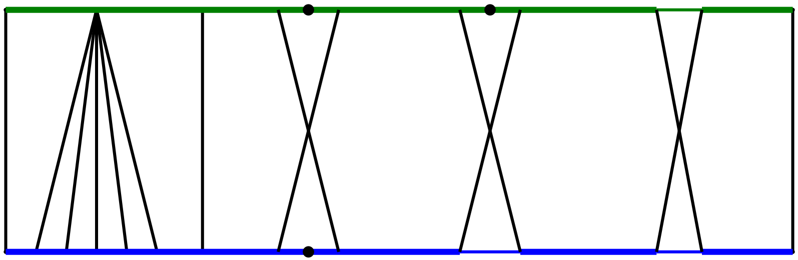


Messy Ladder to Clean Ladder

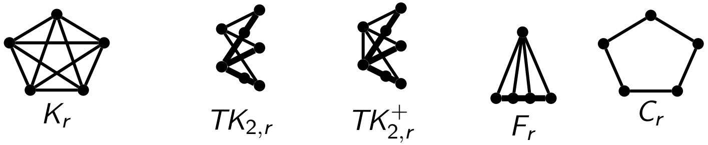
After a series of other observations we get...

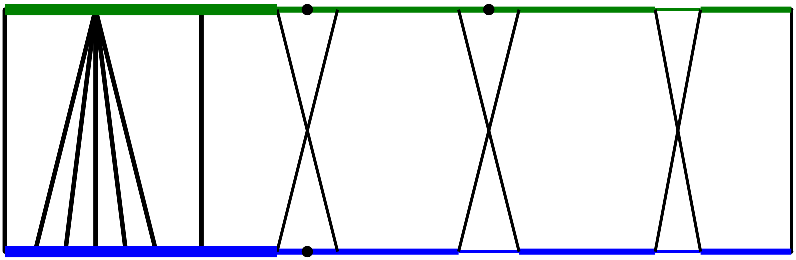
Messy Ladder to Clean Ladder



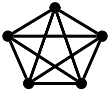


Messy Ladder to Clean Ladder





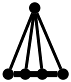
Messy Ladder to Clean Ladder



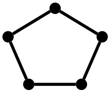
K_r

$TK_{2,r}$

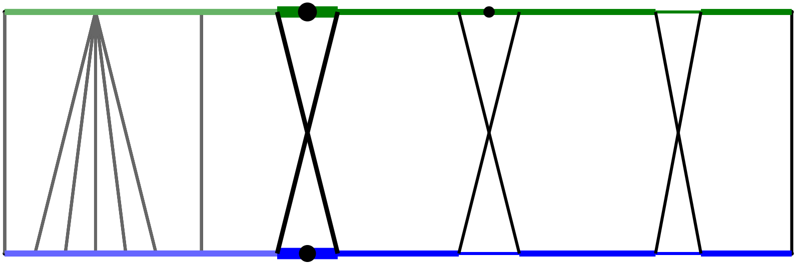
$TK_{2,r}^{+}$



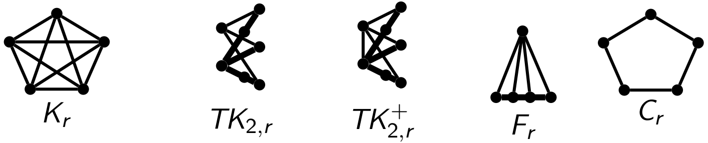
F_r

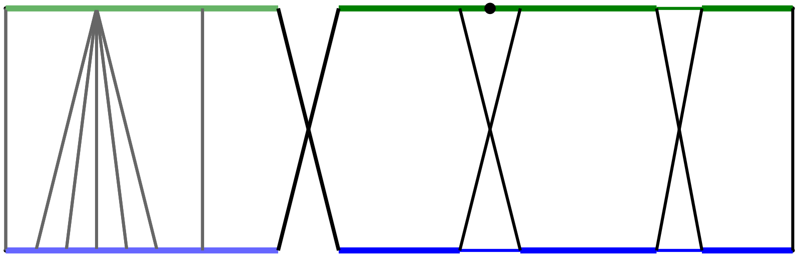


C_r

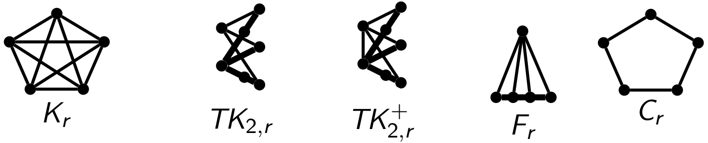


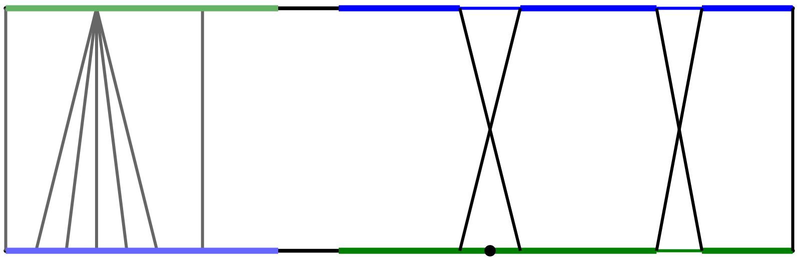
Messy Ladder to Clean Ladder



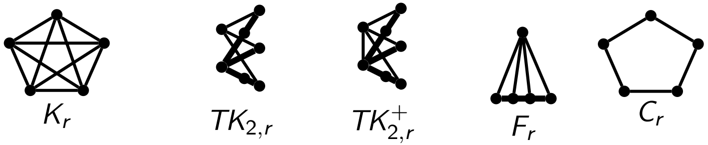


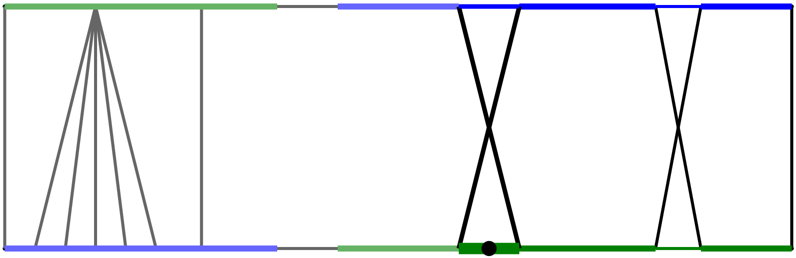
Messy Ladder to Clean Ladder



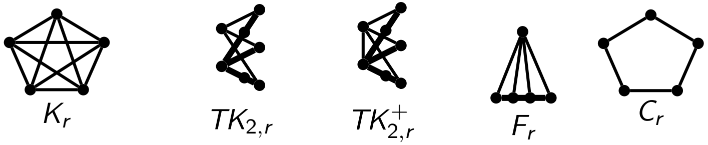


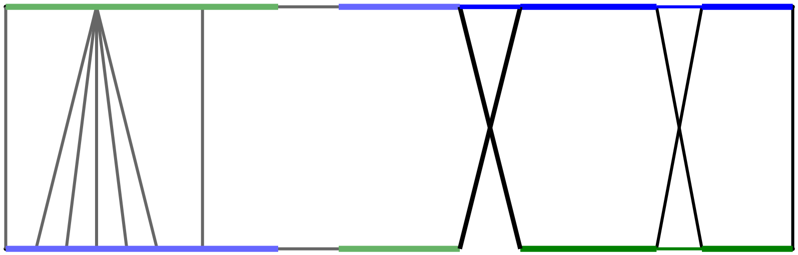
Messy Ladder to Clean Ladder



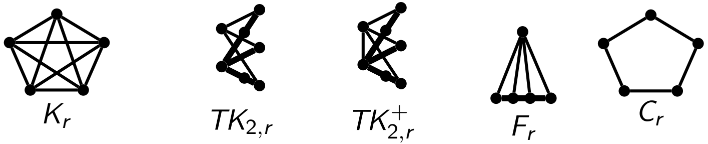


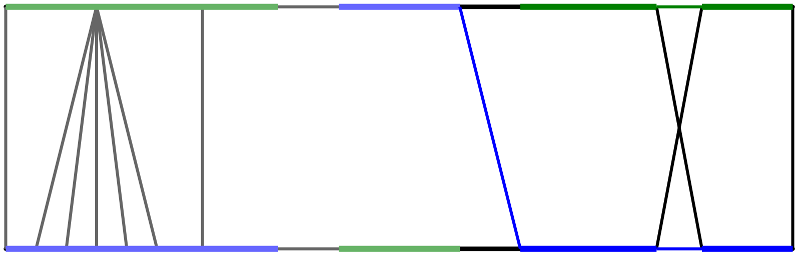
Messy Ladder to Clean Ladder

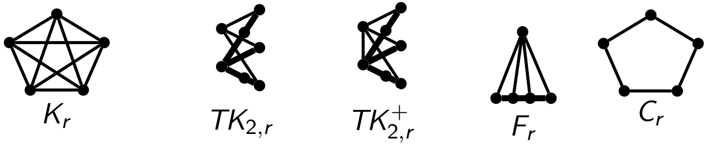


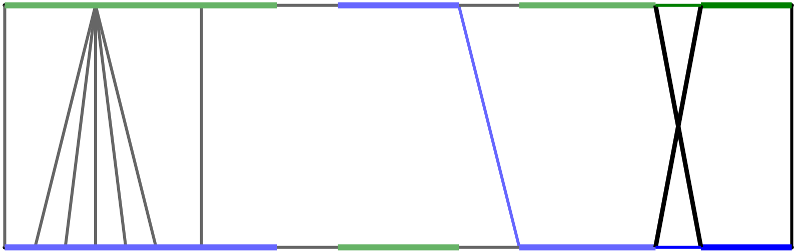


Messy Ladder to Clean Ladder

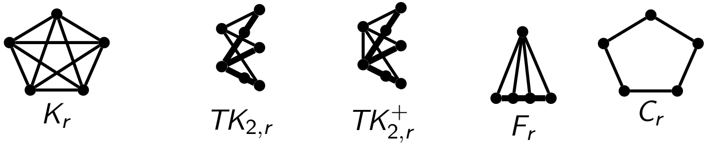


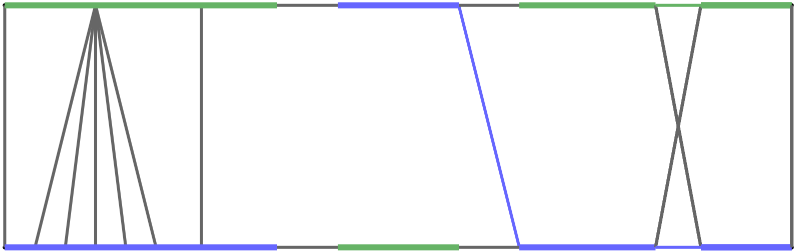



K_r
$TK_{2,r}$
$TK_{2,r}^{+}$
F_r
C_r



Messy Ladder to Clean Ladder

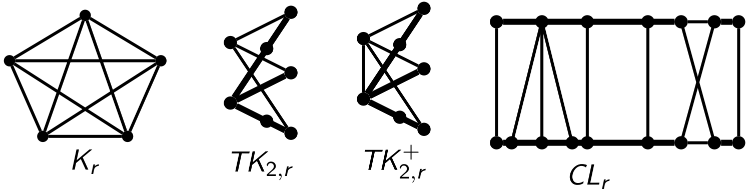




Summary

Theorem (A., D.,O. 2019+)

Every gargantuan 2-connected graph contains a large K_r, $TK_{2,r}$, $TK^+_{2,r}$, or CL_r as an induced subgraph.



What's Next

We are working on cleaning up the result on infinite graphs

Thank you for your attention!

Conjecture (ADO)

Let G be a 2-connected infinite graph that does not contain TK_∞ as a subgraph. Then G contains an induced $TK^+_{2,\infty}$, $TK_{2,\infty}$, TF_∞, a graph obtained from a fan, or one of the following with a ladder type structure.