Automorphisms of Indecomposable Ordered Sets

Bernd Schröder

Ordered Sets, Automorphisms, Endomorphisms

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Not interested? Nap until "Interdependent Orbit Unions."

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The Automorphism Conjecture: Yes.

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What about the number of automorphisms?

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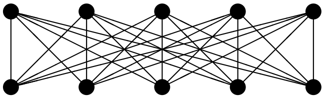

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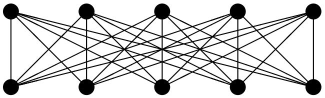

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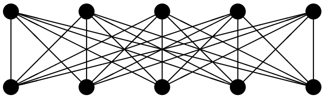

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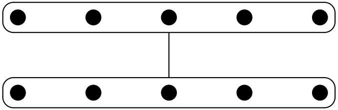

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There's a BS paper that indicates that that stuff most likely can be handled ... but what is that stuff?

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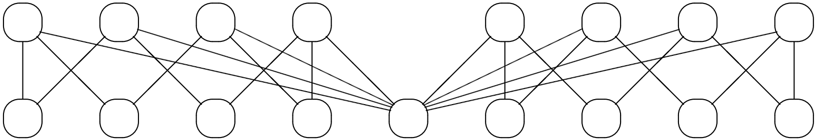

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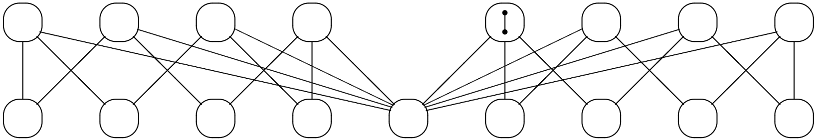

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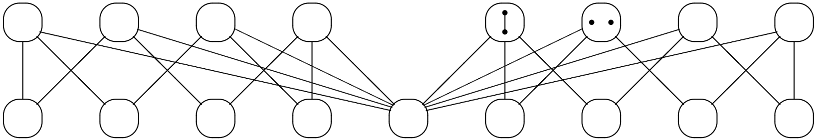

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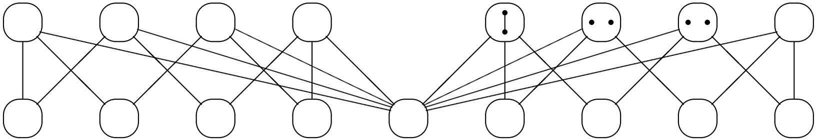

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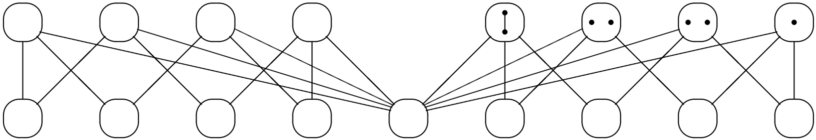

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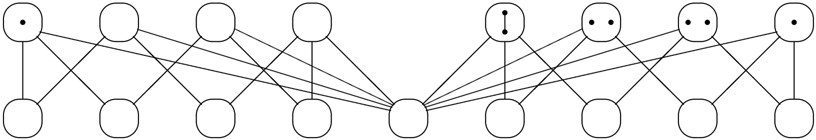

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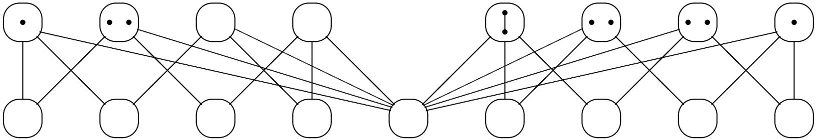

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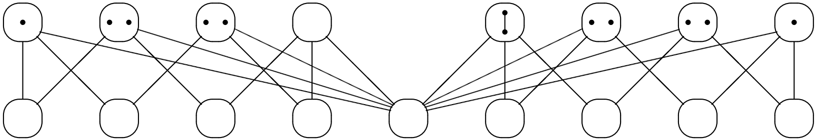

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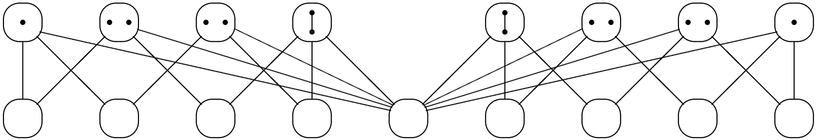

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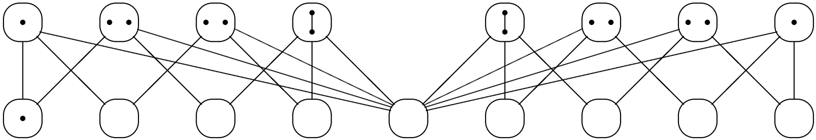

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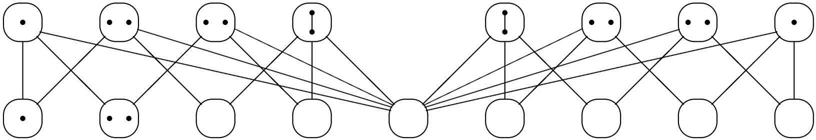

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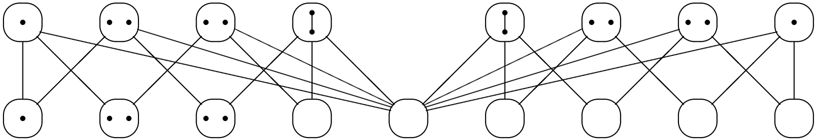

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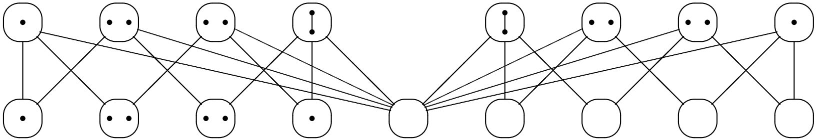

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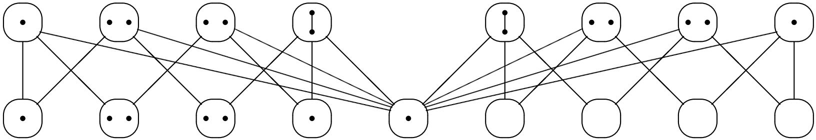

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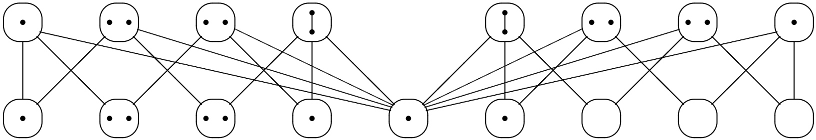

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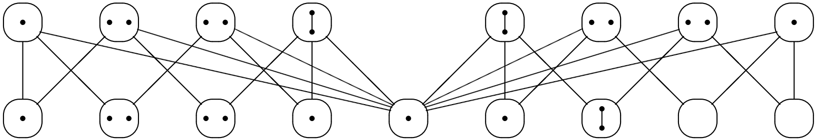

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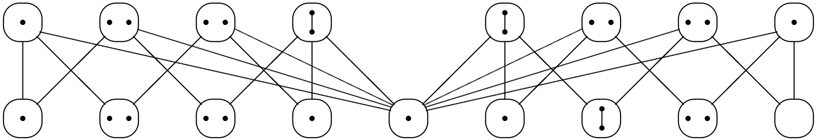

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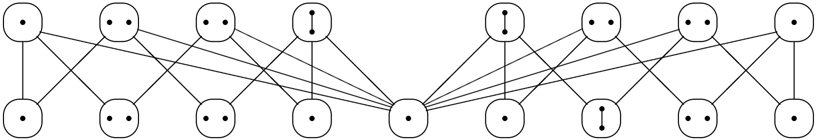

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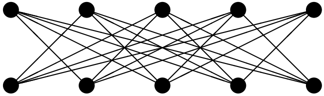

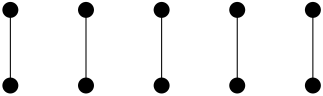

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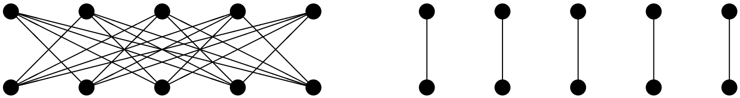

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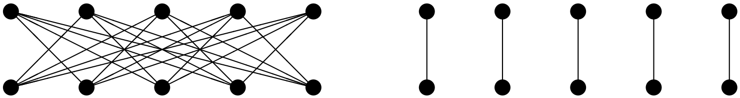

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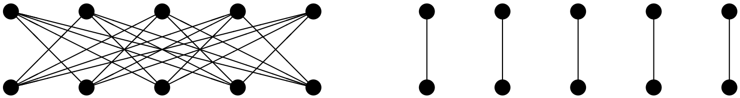

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What is the Largest Number of Automorphisms for an Indecomposable Ordered Set of Width w?

Okay, so I took a walk and started thinking. And the talk so far should have indicated that therein usually lies the problem with me.

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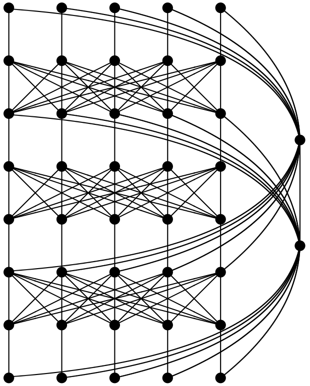

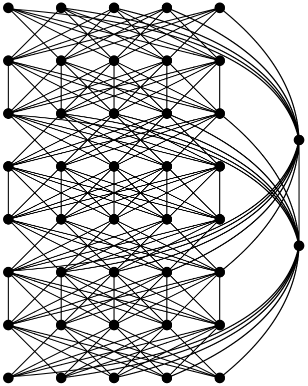

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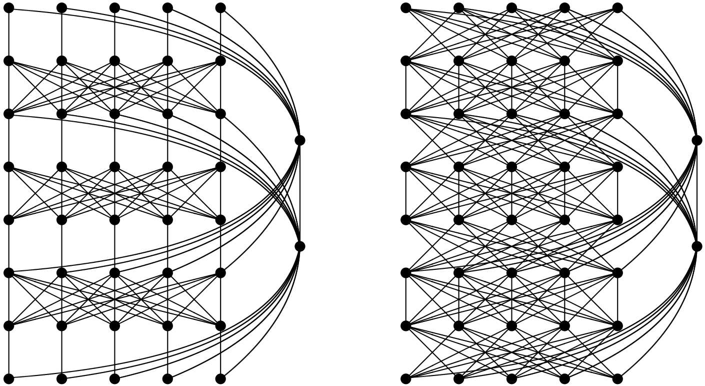

$((w-1)!)^{2\left\lfloor\frac{n}{4w-3}\right\rfloor+2}$ is an upper bound. (Sketch later.)

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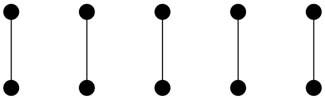

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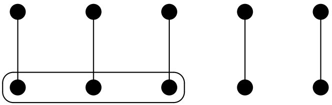

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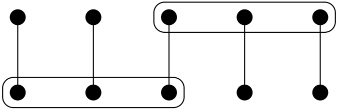

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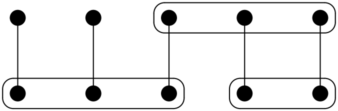

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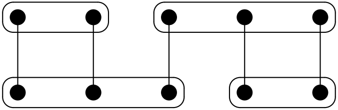

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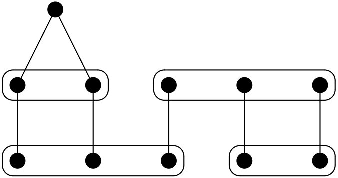

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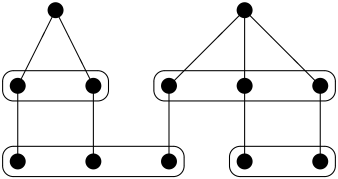

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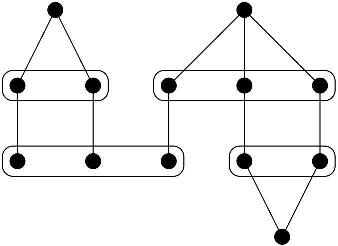

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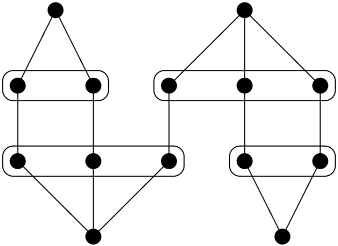

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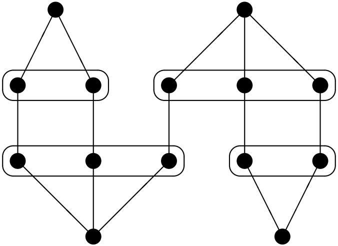

Would "Dictated Orbit Structure" be better?

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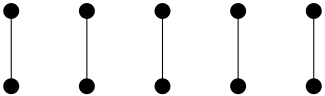

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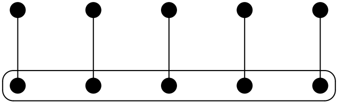

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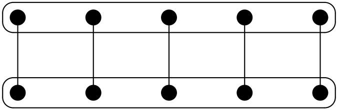

Directly Interdependent Orbits

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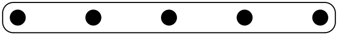

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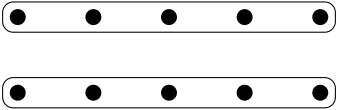

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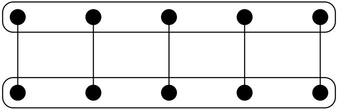

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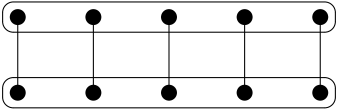

(This can get a lot more complicated.)

Interdependent Orbits

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Two $\mathscr{D}$-orbits C and D with $C \leftrightsquigarrow_{\mathscr{D}} D$ will be called **interdependent**.

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Automorphisms and Interdependent Orbit Unions

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Define $\mathrm{Aut}^P_{\mathscr{D}|U}(U)$ to be the set of automorphisms $\Psi^P \in \mathrm{Aut}(P)$.

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and consequently $|\text{Aut}(P)| = \prod_{j=1}^{z} \left| \text{Aut}^{P}_{\mathscr{N}|U_j}(U_j) \right|$.

Automorphisms and Interdependent Orbit Unions

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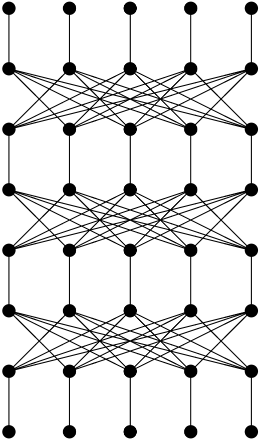

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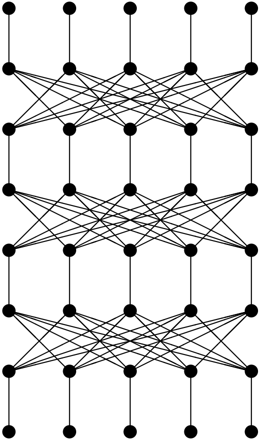
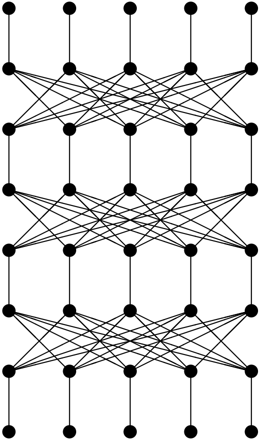


Width w and $> (w-1)!$ permutations

Automorphisms and Interdependent Orbit Unions



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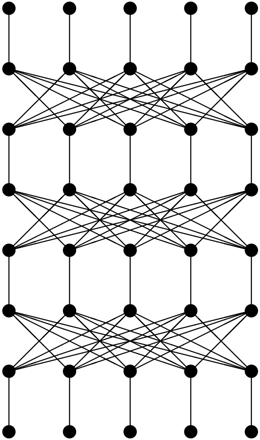
Automorphisms and Interdependent Orbit Unions



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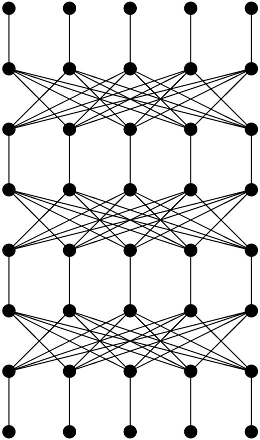

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$> (w-1)^w$ "useable" endomorphisms

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So when I prepared, I really hoped we'd be out of time now ;)